Misunderstandings over Weatherall and Manchak's proof against conventionalism in general relativity

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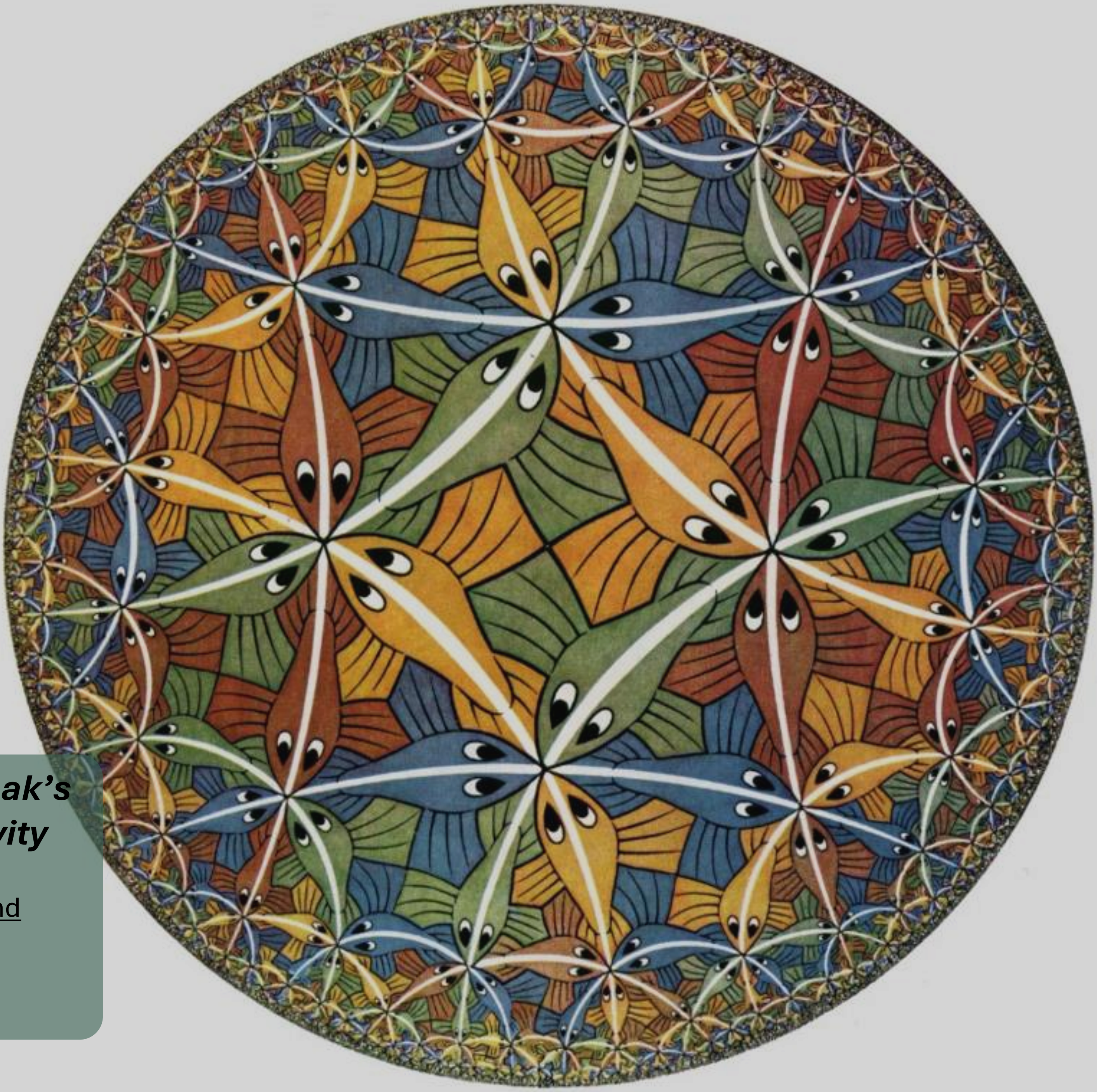

Weatherall & Manchak and Dürr and Ben-Menahem

Existence claims, theorem Θ, torsion

The only genuine assumption: Forces and FORCEs

Conclusion: *le bons sens géométrique*

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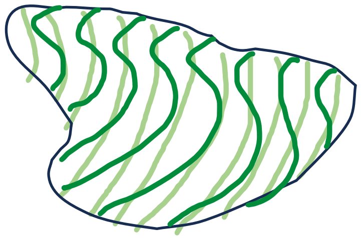
Geometric conventionalism: trading off geometry for forces

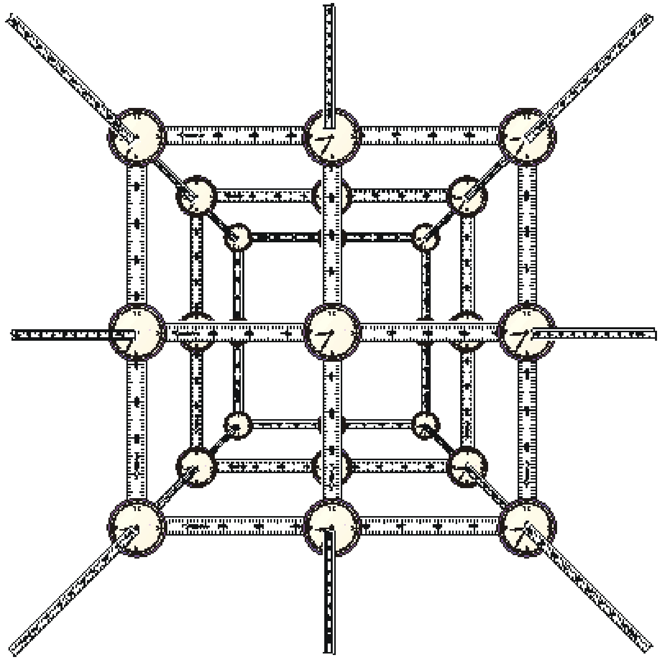
Physical geometry (Helmholtz 1866): there is an empirical question to be asked about the *true geometry* of the world, surveyable by rods and clocks.

Conventionalism about space(time): ascertaining the true geometry of the world requires a conventional choice between empirically-equivalent models.



Spacetime (M, g, ∇)
with dynamics: $\xi^b \nabla_b \xi^a = 0$

Spacetime $\left(M, \tilde{g}, \tilde{\nabla}\right)$
with dynamics: $\xi^b \tilde{\nabla}_b \xi^a + \text{universal force} = 0$



Trade-off equation:

$$a^a = \xi^b \nabla_b \xi^a = \xi^b \tilde{\nabla}_b \xi^a + F_b{}^a\, \xi^b, \text{ for universal “force” } F_{ab} \coloneqq g_{ab} - \tilde{g}_{ab}.$$

Hermann von Helmholtz (1866). “On the Factual Foundations of Geometry.”
Reichenbach (1928). *Raum und Zeit*.

Weatherall & Manchak: "the geometry of conventionality"

Weatherall & Manchak (2014): can these trade-offs *always* be made given the mathematics of our spacetime theories?


Newtonian gravity (NG): yes, the trade-off *is* possible.
General relativity (GR): no, the trade-off is *not* possible.

Striking:
(i) Much-needed rigour to a conceptual debate;
(ii) Explicit theory-dependence.

But also a rather unconventional anti-conventionality!
(iii) Not much conceptual discussion;
(iv) How good are the assumptions?

The relativistic case (Proposition 2.)

Let (M, g_{ab}) be a relativistic spacetime, let $\tilde{g}_{ab} = \Omega^2 g_{ab}$ be a metric conformally equivalent to g_{ab} and let ∇ and $\tilde{\nabla}$ be the Levi-Civita derivative operators compatible with g_{ab} and $\tilde{g}_{ab}$, respectively. Suppose Ω is nonconstant. ***Then there is no tensor field $\boldsymbol{F_{ab}}$*** *such that an arbitrary curve γ is a geodesic relative to ∇ if and only if its acceleration relative to $\tilde{\nabla}$ is given by $F^a{}_n \xi^n$, where ξ^n is the tangent field to γ with unit length relative to $\tilde{g}_{ab}$. (W&M, pp. 242-3)*

Jim Weatherall & J.B. Manchak (2014). "The Geometry of Conventionality."

Dürr & Ben-Menahem's response: pointing out loopholes

Dürr & Ben-Menahem (2022):

→ list all the assumptions;
→ frame the result as an **inconcistency proof**;
→ and argue *all* of them are "unwarranted";
→ thus "short-circuiting" and "by-passing" the proof;
→ instead adopt a selective anti-realism (i.e. conventionalism).




Wait what? Was it all for nothing?

The rest of this talk:

I. <u>Technical:</u>
- The assumptions;
- The target is universality, not existence;
- Thus there is no interesting inconsistency proof;
- Illustrated via generalisation to torsionful spacetime;

II. <u>Conceptual:</u>
- The force concept as a *universal standard force*;
- Responding to underdetermination via *le bon sens géométrique.*

Patrick Dürr & Yemima Ben-Menahem (2022).
"Why Reichenbach wasn't entirely wrong, and Poincaré was almost right,…."

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Proofs make assumptions

Dürr and Ben-Menahem's list of assumptions:

(FORCE). A compensating universal force should be a 2-tensor F_{ab}.
(CONF). The alternative metric is conformally related to the standard metric: $\tilde{g}_{ab} = \Omega^2 g_{ab}$.
(NORM). $\tilde{\xi}^b$ is a vector of unit norm with respect to the new geometry: $\tilde{g}_{ab}\tilde{\xi}^a\tilde{\xi}^b = 1$.
(RIEM). Geometric alternatives must employ pseudo-Riemannian geometry.

I highlight: - **(RIEM-SYMM).** The affine connection is symmetric.
- **(RIEM-COMP).** The affine connection is metric compatible.

I add: - **(DIM4).** We work in 3+1 dimensions.

Dürr & Ben-Menahem (and Tasdan & Thébault, 2024):
(ALT-ACC). "Geometric alternatives for general-relativistic acceleration of a test-particle, $\xi^b \nabla_b \xi^a$ must *exist*".

This forms a *set of inconsistent premisses.* A "no-go theorem".

Rejecting a premisse → interesting philosophical position.

Patrick Dürr & Yemima Ben-Menahem (2022). "Why Reichenbach wasn't entirely wrong, and Poincaré was almost right,…."
Ruward Mulder (2024). "Relativity and conventionality: Reichenbach's theorem θ and Weatherall & Manchak's proof against it."
Ufuk Tasdan & Karim Thébault (2023). "Spacetime Conventionalism Revisited."

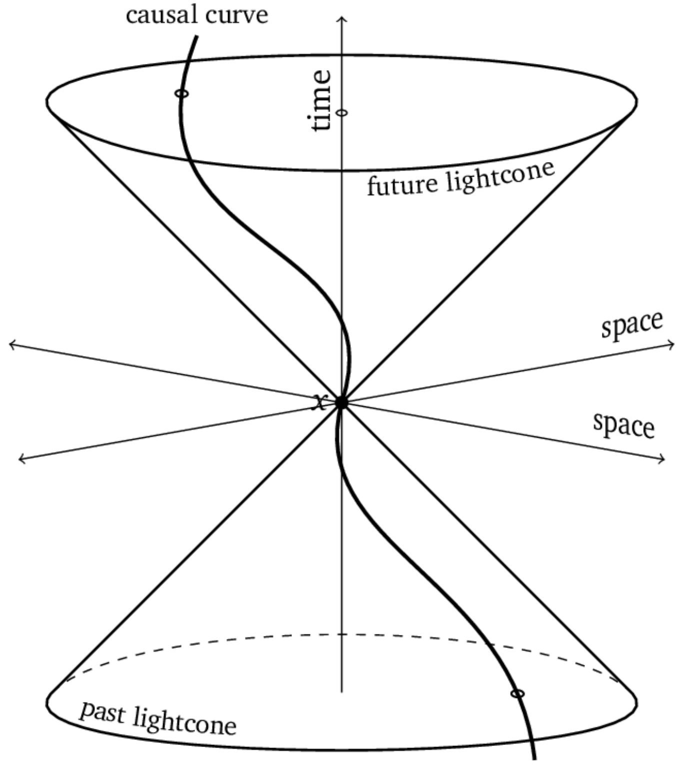
(CONF) Restriction to conformally equivalent spacetimes

At first sight, this seems the way the go: we should be allowed to look *everywhere* for an empirically equivalent geometry!

Yet, W&M present the restriction to conformal spacetimes (**CONF**) as a good thing:

> *"Note, though, that requiring conformal equivalence only strengthens our results. If the conventionalist cannot accommodate conformally equivalent metrics, then a fortiori one cannot accommodate arbitrary metrics; conversely, if Reichenbach's proposal fails even in the special case of conformally equivalent metrics, then it fails in the case of arguably greatest interest."* (W&M, p. 237)

To see what is going on, I think we should go back to Reichenbach's formulation.



Reichenbach's theorem Θ: a universality claim

Theorem Θ:
*Given a [Euclidean] geometry G0 to which the measuring instruments conform, we can imagine a universal force F [i.e. $\boldsymbol{F^a}_{\boldsymbol{b}}$, under (**FORCE**)] which affects the instruments in such a way that the actual geometry is an arbitrary geometry G, while the observed deviation from G is due to a universal deformation of the measuring instruments. (Reichenbach, p. 33)*

Theorem Θ is radical! → it is a universality claim:

Any metric g can then reproduce the same empirical content as *any other* metric g̃ !

Theorem Θ is W&M's indirect target

But the negation of a universality claim remains true under generalisation!

→ There is no inconsistency proof!

Reichenbach (1928). *Raum und Zeit.*

Universality vs. Existence: why there is no interesting inconsistency proof

(Conv $-\ \forall\forall$) or "theorem ϴ": *Any* metric can equally represent the empirical content as *any* other metric, if F_{ab} exists.

(Conv $-\forall_{conf}\forall_{conf}$): *Any* two conformally equivalent metrics can equally represent the empirical content, if F_{ab} exists (W&M's target).

- W&M show **~(Conv-$\forall_{conf}\forall_{conf}$) ;**
- (**Conv-$\forall_{conf}\forall_{conf}$**) → (**Conv—∀∀**) **;** (because it is *not* an existence claim!)
- Thus ~**(Conv—∀∀)** (by *modus tollens*).

But this is <u>**not**</u> what modern conventionalists have in mind!

(Conv—∀∃): For *any* metric, there *exists* another metric that can equally represent the empirical content.

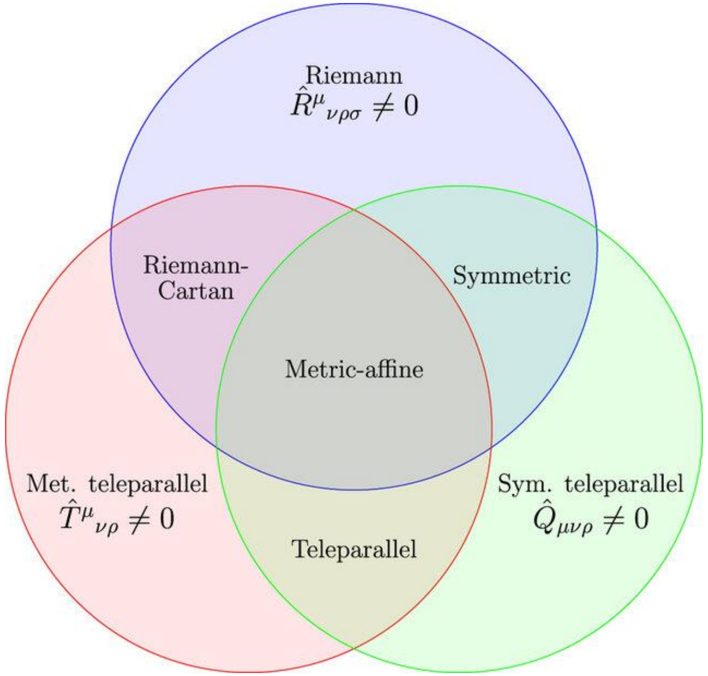
Dürr & Ben-Menahem's **(ALT-ACC)**:
"Geometric alternatives for general-relativistic acceleration of a test-particle, $\xi^b\nabla_b\xi^a$ must *exist*"…
… is a misreading of the proof as a rejection of **(Conv—∀∃)** instead of **(Conv—∀∀).**

→ The only genuine loophole is (**FORCE**) !

Illustration and generalisation: rejecting **(RIEM-SYMM)**

There exist viable non-Riemannian theories: the Geometric Trinity of Gravity.

Let's include teleparallel gravity by allowing for not-necessarily-symmetric spacetimes:



<u>The torsionful relativistic case</u> (Proposition 3.)

Let (M, g_{ab}) be a relativistic spacetime, let $\tilde{g}_{ab} = \Omega^2 g_{ab}$ be a metric non-trivially conformally equivalent to g_{ab} and let ∇ and $\tilde{\nabla}$ be **not-necessarily-symmetric** *derivative operators compatible with g_{ab} and and $\tilde{g}_{ab}$, respectively.* ***Then there is no tensor field $\boldsymbol{F_{ab}}$*** *such that an arbitrary curve γ is a geodesic relative to ∇ if and only if its acceleration relative to $\tilde{\nabla}$ is given by $F^a{}_n\tilde{\xi}^n$, where $\tilde{\xi}^n$ is the tangent field to γ with unit length relative to $\tilde{g}_{ab}$. (Mulder 2024, p. 14)*

The proof goes through in a fully analogous manner, since the contorsion tensor $K^a{}_{bc}$ is independent of the metric! Jensen (2005): $C^a{}_{bc} = \frac{1}{2} g^{an}(\nabla_n g_{bc} - \nabla_b g_{nc} - \nabla_c g_{bn}) + K^a{}_{bc}$.

Weatherall & Manchak's proof *generalises* under the rejection of (**RIEM-SYMM**).

Jiménez Beltrán, Heisenberg L, Koivisto TS. (2019). "The Geometrical Trinity of Gravity."
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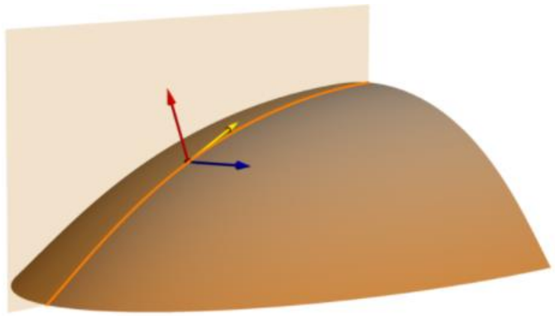
Forces and FORCEs

(FORCE). A compensating universal force should be a 2-tensor F_{ab}.

> [...] we believe that any reasonable account of "force" or "force field" in a Newtonian or relativistic framework would need to agree on at least this much, and so when we refer to forces/force fields "in the standard sense," we have in mind forces or force fields that have the character we describe here. (W&M, fn. 5, p. 235), p. 14)



- (**FORCE-a**) a force is some physical quantity acting on a massive body or point particle;
- (**FORCE-b**) forces are represented by vectors at a point (at least in Newtonian gravity and general relativity);
- (**FORCE-c**) the total force acting on a particle at a point must be proportional to the acceleration of the particle at that point, and vanishes just in case the acceleration vanishes.

Physical interpretation: pulling you off a geodesic, acceleration tangent to the curve, i.e. a 2-tensor.

Let us call this the "standard force".

Dürr & Ben-Menahem against (**FORCE**)

D&BM say (**FORCE**) is overly restrictive because it is too conservative.

Why not consider a broader notion of ‘interaction’, including 3-tensors $G^a{}_{bc}$?

1) The force concept has historically proven to be considerably variable (Jammer 1957, Hesse 1961):

> “to ennoble conservativeness as an *unqualified* virtue per se, we regard as unduly reactionary [...]; such a view would be amply belied by the history of science.” (D&BM 2022, p. 163).

2) Nor can one appeal to simplicity of tensor rank as a criterion for theory choice:

> Simplicity “is a pragmatic maxim par excellence (see e.g. Bunge, 1963). Its connection with truth is evidently fragile. It has its place as a heuristic rule of thumb in theory construction and assessment in praxi: it recommends to first inspect conservative hypotheses, which fit in most easily with our background assumptions, before considering more radical ideas.” (D&BM, p. 163)

Max Jammer, (1957). *Concepts of Force*.

Mary B. Hesse, (1961). *Forces and Fields: The Concept of Action at a Distance in the History of Physics*.

Ruward Mulder (2024). “Relativity and conventionality: Reichenbach’s theorem θ and Weatherall & Manchak’s proof against it.”

No universal standard force equivalent to GR

A universal standard force also has a *comparative* function.

Is there a force theory in flat space, equivalent to GR, using *Newtonian* forces?

Newtonian gravitational forces are like universal forces (Dieks 1987), because they
(a) affect all materials in the same way;
(b) cannot be screened-off by insulating walls.

Newton-Cartan theory has causally efficacious space and time …
… and is equivalent to Newtonian gravity, which is a a (**FORCE**)-theory.

	Geometric theory	FORCE theory
Non-relativistic	Newton-Cartan theory	Newtonian gravity
Relativistic	General relativity	???

One would have to prove (or disprove):

Conv-∀g∃η: for each metric g, the Minkowski metric η is capable of reproducing the same observable consequences, given force tensor F_{ab}. (Mulder, p. 17)

(Not likely to be true, for sociological reasons.)

Dennis Dieks (1987). ``Gravitation as a universal force."
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An olive branch: “interesting” and “non-trivial” alternatives

Both sides in this debate search for a conceptual middle ground about the force concept

W&M do *not* rely on ‘simplicity’ *simpliciter*, but simply take an anti-skeptical stance:

“at some abstract level of description, a thesis [i.e. conventionalism] is irrefutable. But at that same level of abstractness, as has often been observed, it is also ***uninteresting****. We can be conventionalists about geometry, perhaps, but in the same way that* ***we could be conventionalists about anything****.”“ (W&M, p. 234)*

… if one is willing to postulate enough—an evil demon, say.” (W&M, p. 246)

D&BM reject of *trivial* semantic holism (this is why Reichenbach is not entirely wrong, but still wrong):

“Reasonable alternatives *to these assumptions exist that open up geometric alternatives [...]*
trivial semantic conventionalism*—trafficking in the conventionality of merely representational/linguistic differences of synonymous/logically equivalent content—is of little relevance to the debate.” (D&BM, p. 170)*

Both sides can be seen as reacting to underdetermination via *le bon sens géométrique*!

Le bon sense (géométrique)

Bon sens is a *supra-empirical response* to underdetermination based on expert judgement.

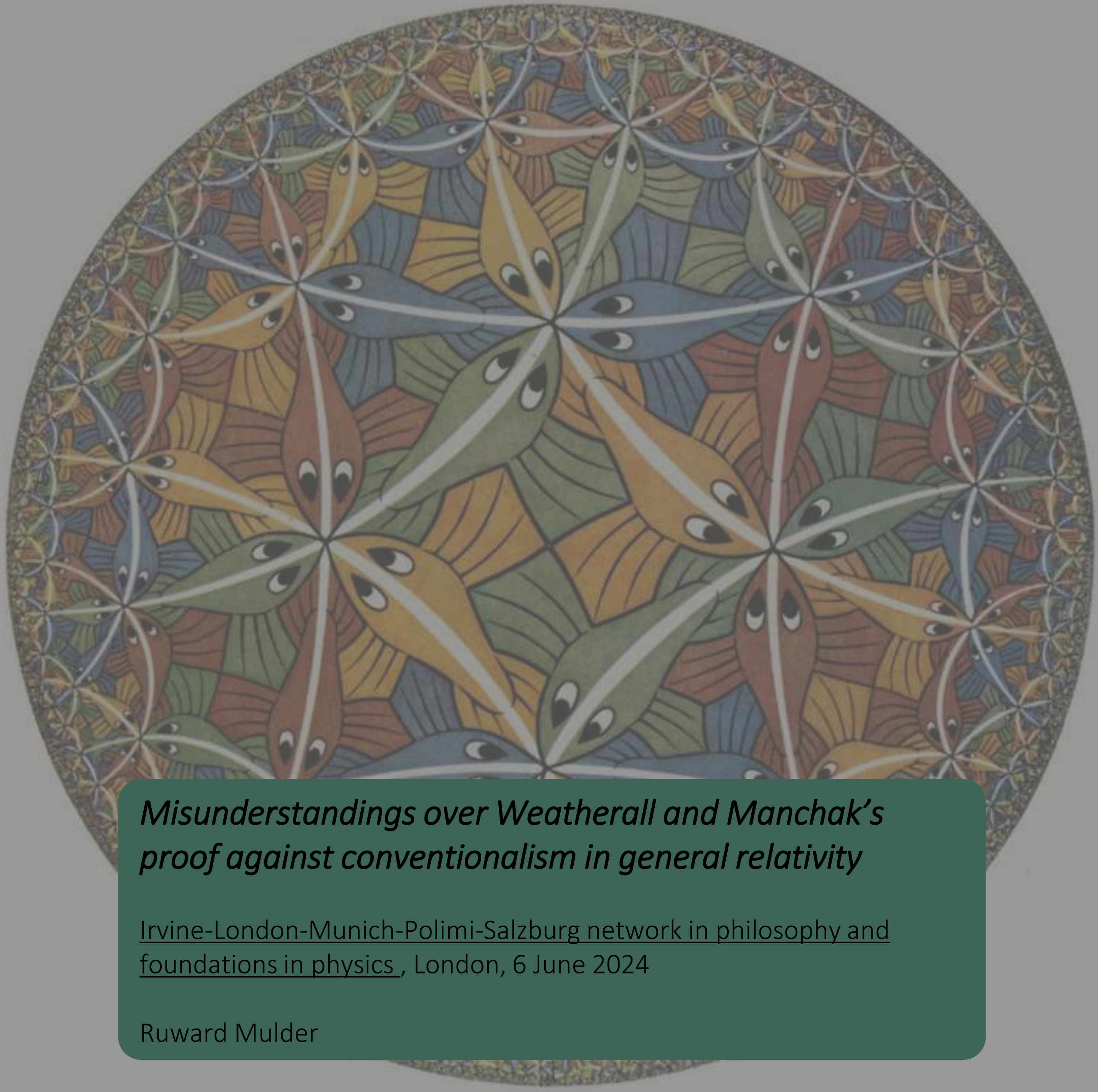
Neither logic not empirical investigation can jully justify a choice.

But that does not mean there is nothing we can do.

> For it may be that good sense permits us to decide between two physicists. [...] But these reasons of good sense do not impose themselves with the same implacable rigor that the prescriptions of logic do. There is something vague and uncertain about them; they do not reveal themselves at the same time with the same degree of clarity to all minds. Hence, the possibility of lengthy quarrels [...]. (Duhem 1906, p. 217)

How far are ***you*** willing to go to cook up alternative mathematical constructions with the same empirical substructure as GR?

Pierre Maurice Marie Duhem (1906). *The Aim and Structure of Physical Theory*.
Milena Ivanova (2010). “Pierre Duhem’s Good Sense as a Guide to Theory Choice”.



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Dieks, Dennis (1987). ``Gravitation as a universal force." *Synthese* **73**. Online access.

Duhem, Pierre Maurice Marie (1906). *The Aim and Structure of Physical Theory*. Princeton, 1954: Princeton University Press.

Dürr, Patrick & Yemima Ben-Menahem (2022). "Why Reichenbach wasn't entirely wrong, and Poincaré was almost right, about geometric conventionalism." *Studies in History and Philosophy of Science Part A* **96** (C): pp. 154–173. Online access.

von Helmholtz, Hermann (1866). "On the Factual Foundations of Geometry." In *Beyond Geometry. Classical Papers from Riemann to Einstein.* Edited by Peter Pesic. Dover

Hesse, Mary B. (1961). *Forces and Fields: The Concept of Action at a Distance in the History of Physics*. Mineola, N.Y.: Dover Publications.

Ivanova, Milena (2010). "Pierre Duhem's Good Sense as a Guide to Theory Choice*". Studies in History and Philosophy of Science Part A* **41.1**, pp. 58–64. Online access.

Jammer, Max (1957). *Concepts of Force*. Dover Publications (1999).

Jensen, Steuard (2005). "General Relativity with Torsion: Extending Wald's Chapter on Curvature". *Lecture notes*. Online access.

Jiménez Beltrán, Heisenberg L, Koivisto TS. (2019). "The Geometrical Trinity of Gravity." *Universe*.; 5(7):173. Online access.

Mulder, Ruward A. (2024). "Relativity and conventionality: Reichenbach's theorem θ and Weatherall & Manchak's proof against it." Chapter 4 of *PhD Dissertation.*
Available upon request.

Reichenbach, Hans (1928, 2014). *The Philosophy of Space & Time*. Dover Publications.

Tasdan, Ufuk I & Karim P.Y. Thébault (2023). "Spacetime Conventionalism Revisited." *Philosophy of Science* **91** (2), pp. 488–50. Online access.

Weatherall, Jim & JB Manchak (2014). "The Geometry of Conventionality." *Philosophy of Science* **81** (2), pp. 233-247. Online access.

No one here is arguing about conventionalism!

Conventionalism is:
- A response to underdetermination
- A common core approach: selective realism about the formalism, denying truth value of parts of the formalism.

There is no dispute over truth values or the nature of the convention.

The debate is not over conventionalism
→ but over whether GR is safe for conventionalism.

Given a theory, are these trade-offs between force and geometry possible?

It is important whether this is UDD of models or UDD of theories.
→ Both are possible!