Relativistic holism and the demon tensor: is theorem theta a no-go?

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Geometric holism by universal effects

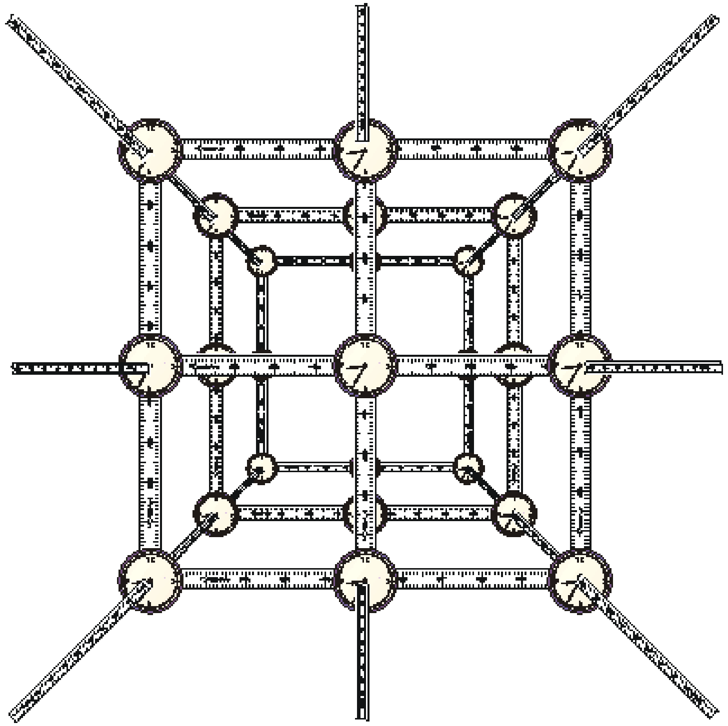
Weatherall & Manchak's proof and its assumptions

Existence and universality, theorem Θ, and generalization

Forces and FORCEs: tensor rank simplicity?

Universal “forces”: 100 years of trading off geometries (1/3)

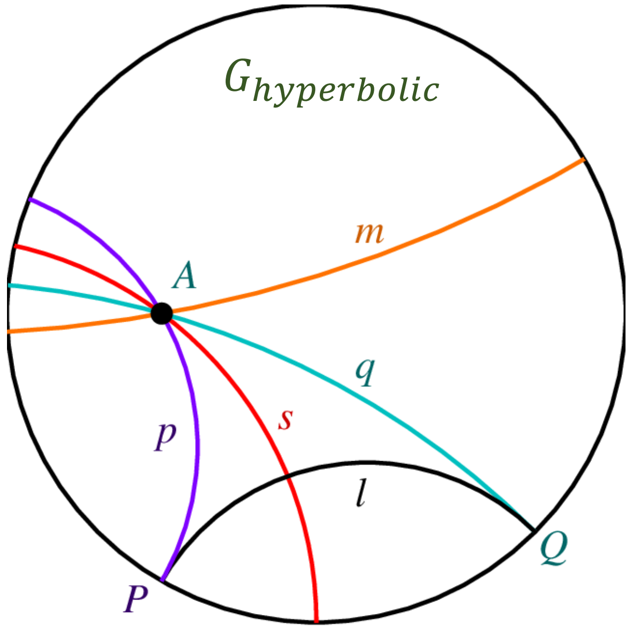
Physical geometry (Helmholtz 1866): empirical question for the *physical geometry* of the world: rods and clocks.



Universal “forces”: 100 years of trading off geometries (2/3)

Physical geometry (Helmholtz 1866): empirical question for the *physical geometry* of the world: rods and clocks.

Geometric holism based on Poincaré’s equivalent proofs (1891): there are many empirically equivalent combinations of geometries [G] and “universal forces” (“universal effects”) [F]: {G+F , G’+F’ , G’’+F’’, ... }.



Universal "forces": 100 years of trading off geometries (3/3)

Physical geometry (Helmholtz 1866): empirical question for the *physical geometry* of the world: rods and clocks.

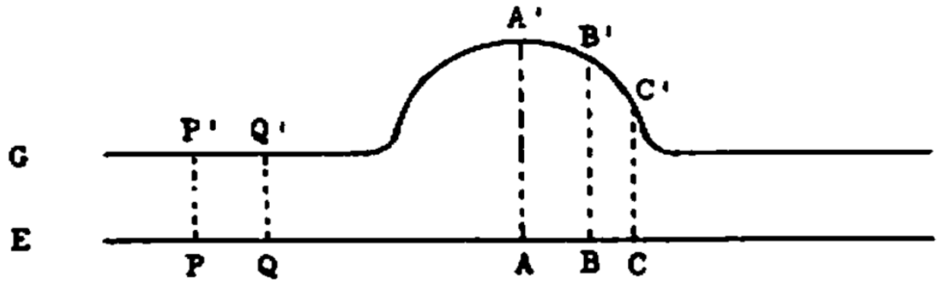
Geometric holism based on Poincaré's equivalent proofs (1891): there are many empirically equivalent combinations of geometries [G] and "universal forces" ("universal effects") [F]: {G+F , G'+F' , G''+F'', … }.

Conventionalism about space(time): ascertaining the physical geometry requires (in some way) a conventional choice.
→ Loosely associated with Poincaré, Duhem, Schlick, Carnap, and others, but mostly Reichenbach (1926, Sec. 8).:

Theorem θ: "Given a geometry *G'* to which the measuring instruments conform, we can imagine a universal force *F* which affects the instruments in such a way that the actual geometry is an arbitrary geometry *G*, while the observed deviation from *G* is due to a universal deformation of the measuring instruments."[1]

Theorem *θ* shows all geometries to be equivalent; it formulates the *principle of the relativity of geometry*. It follows that it is meaningless to speak about one geometry as the *true* geometry.

An effect (force) is **universal** iff it
1. cannot be screened off by insulating walls
2. acts equally on all materials/particle species
Otherwise it is a **differential** effect (or force)



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- Forces and FORCEs: tensor rank simplicity?

Universal forces in relativistic spacetimes

GR models: manifold M equipped with metric tensor g^{ab} and associated (implicitly defineable) Levi-Civita connection ∇^a.

Construct a distinct geometry on M:
→ take any other metric $\tilde{g}_{ab}$, equipped with $\tilde{\nabla}^a$.

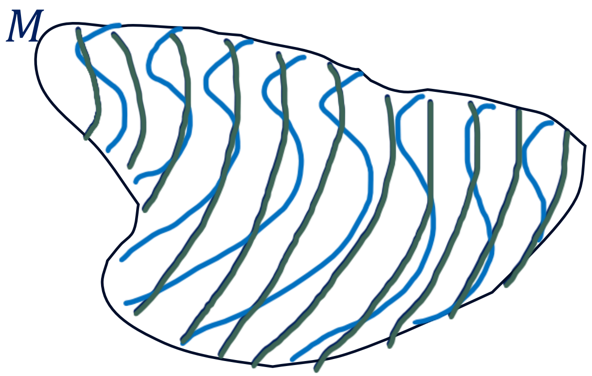
Then construct $(\mathbf{F})$ such that they correct for the difference between the affine-geodesics of the new geometry with the affine-geodesics of the old.

Does this work for $F_{ab} \coloneqq g_{ab} - \tilde{g}_{ab}$?
Sure! → connection coefficients $\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}(\partial_\mu g_{\nu\rho} + \partial_\mu g_{\nu\rho} - \partial_\mu g_{\nu\rho})$
become:

$$\tilde{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} + \frac{1}{2} F^{\rho\sigma}(\partial_\mu g_{\nu\rho} + \partial_\mu g_{\nu\rho} - \partial_\mu g_{\nu\rho}) + \frac{1}{2} g^{\rho\sigma}(\partial_\mu F_{\nu\rho} + \partial_\mu F_{\nu\rho} - \partial_\mu F_{\nu\rho}) + \frac{1}{2} F^{\rho\sigma}(\partial_\mu F_{\nu\rho} + \partial_\mu F_{\nu\rho} - \partial_\mu F_{\nu\rho}).$$

So the universal effect manifests as a rank-3 tensor field.
→ That's an odd "force"!

Spacetime (M, g, ∇)
with geodesics: $\xi^b \nabla_b \xi^a = 0$



Spacetime $(M, \tilde{g}, \tilde{\nabla})$
with geodesics: $\tilde{\xi}^b \tilde{\nabla}_b \tilde{\xi}^a = 0$

Spacetime $(M, \tilde{g}, \tilde{\nabla})$
with dynamics: $\tilde{\xi}^b \tilde{\nabla}_b \tilde{\xi}^a + (\mathbf{F}) = 0$

Weatherall & Manchak: “the Geometry of Conventionality”

Weatherall & Manchak (2014): can these trade-offs *always* be made given rigorous formulations of our spacetime theories?

Newtonian gravity (NG): yes, the trade-off *is* possible.
General relativity (GR): no, the trade-off is *not* possible.

Striking:
(i) Much-needed rigour to a conceptual debate;
(ii) Explicit theory-dependence.

But also a rather unconventional anti-conventionality!
(iii) Little engagement with Reichenbach’s project;
(iv) How good are the assumptions?

The relativistic case (Proposition 2.)

Let (M, g_{ab}) be a relativistic spacetime, ***let $\tilde{g}_{ab} = \Omega^2 g_{ab}$*** *be a metric conformally equivalent to g_{ab} and let ∇ and $\tilde{\nabla}$ be the Levi-Civita derivative operators compatible with g_{ab} and $\tilde{g}_{ab}$, respectively. Suppose Ω is nonconstant.* ***Then there is no tensor field F_{ab}*** *such that an arbitrary curve γ is a geodesic relative to ∇ if and only if its acceleration relative to $\tilde{\nabla}$ is given by $F^a{}_n \tilde{\xi}^n$, where $\tilde{\xi}^n$ is the tangent field to γ with unit length relative to $\tilde{g}_{ab}$. (W&M, pp. 242-3)*

Proofs make assumptions

Dürr and Ben-Menahem's list of assumptions:

(FORCE). A compensating universal force should be a 2-tensor F_{ab} in the geodesic equation.

(CONF). The alternative metric is conformally related to the standard metric: $\tilde{g}_{ab} = \Omega^2 g_{ab}$.

(NORM). $\tilde{\xi}^b$ is a vector of unit norm with respect to the new geometry: $\tilde{g}_{ab}\tilde{\xi}^a\tilde{\xi}^b = 1$.

(RIEM). Geometric alternatives must employ pseudo-Riemannian geometry.

I highlight:
- **(RIEM-SYMM)**. The affine connection is symmetric.
- **(RIEM-COMP)**. The affine connection is metric compatible.

I add:

(TOPO): Geometric alternatives must be constructed on the same manifold

In particular:
- **(DIM4)**. The manifold has 3+1 dimensions.
- **(HAUS)**. Points can be kept apart by open sets.

Details of the proof (see Hand-out)

Geodesic equation $a^a = \tilde{\xi}^b \widetilde{\nabla}_b \tilde{\xi}^a = \tilde{\xi}^b \nabla_b \tilde{\xi}^a + C^a_{\ bc} \tilde{\xi}^a \tilde{\xi}^b$, for $\nabla = (\widetilde{\nabla}, C)$, to compute ((**RIEM**) & (**CONF**), $\tilde{g}_{bc} = \Omega^2 g_{bc}$):

$C^a_{\ bc} = \frac{1}{2} g^{an}(\nabla_n g_{bc} - \nabla_b g_{nc} - \nabla_c g_{bn}) = C^a_{\ bc} = \frac{1}{2\Omega^2}(g_{bc} g^{an} \nabla_n \Omega^2 - \delta_c^{\ a} \nabla_b \Omega^2 - \delta_b^{\ a} \nabla_c \Omega^2)$.

Use (**NORM**): $g_{ab}\, \xi^a \xi^b = 1 = \tilde{g}_{ab} \tilde{\xi}^a \tilde{\xi}^b = g_{ab} \Omega^2 \tilde{\xi}^a \tilde{\xi}^b \rightarrow \tilde{\xi}^a = \Omega^{-1} \xi^a$, so that $\tilde{\xi}^b \widetilde{\nabla}_b \tilde{\xi}^a = \cdots = \cdots = \frac{1}{\Omega^3}(\xi^b \xi^c - g^{an}) \nabla_n \Omega$.

If F_{ab} exists that satisfies (**FORCE**) then $\tilde{\xi}^b \widetilde{\nabla}_b \tilde{\xi}^a = F^a_{\ m} \tilde{\xi}^m = \frac{1}{\Omega} \tilde{g}^{an} F_{nm} \xi^m = \frac{1}{\Omega^3}(\xi^b \xi^c - g^{an}) \nabla_n \Omega$. (on a ∇-geodesic.)

Take three timelike geodesics: μ^a, η^a, and $\zeta^a = \alpha(\mu^a + \eta^a) \neq 0$ at p. Each satisfy the force law at point p:

$\tilde{g}^{an} F_{nm} \zeta^m = \frac{1}{\Omega^2}(\zeta^b \zeta^c - g^{an}) \nabla_n \Omega = \frac{1}{\Omega^2}\left(\mu^a \mu^n + \eta^a \mu^n + \mu^a \eta^n + \eta^a \eta^n - \frac{g^{an}}{\alpha^2}\right) \nabla_n \Omega.$

$\tilde{g}^{an} F_{nm} \zeta^m = \alpha\, \tilde{g}^{an} F_{nm} \mu^m + \tilde{g}^{an} F_{nm} \eta^m) = \frac{\alpha}{\Omega^2}(\mu^b \mu^c + \eta^b \eta^c - 2 g^{an}) \nabla_n \Omega.$

Equating and rearranging: $(2\alpha - 1) g^{an} \nabla_n \Omega = -\alpha[(\alpha - 1)(\mu^a \mu^n + \eta^a \eta^n) + \eta^a \mu^n + \mu^a \eta^n] \nabla_n \Omega.$

This cannot be satisfied for arbitrary vectors.
→ Thus there exists no F_{ab} that satisfies (**FORCE**).

Dürr & Ben-Menahem's response: pointing out loopholes

Dürr & Ben-Menahem (2022) add:

(ALT-ACC). "Geometric alternatives for general-relativistic acceleration of a test-particle, $\xi^b \nabla_b \xi^a$ must exist".

Forms a *set of inconsistent premises* or "no-go theorem":
¬((**ALT-ACC**) ∧ (**CONF**) ∧ (**NORM**) ∧ (**RIEM**) ∧ (**TOPO**) ∧ (**FORCE**));

Equivalent to
¬(**ALT-ACC**) ∨ ¬(**CONF**) ∨ ¬(**NORM**) ∨ ¬(**RIEM**) ∨ ¬(**TOPO**) ∨ ¬(**FORCE**).

For each premise, denying it opens up a potentially interesting philosophical position.

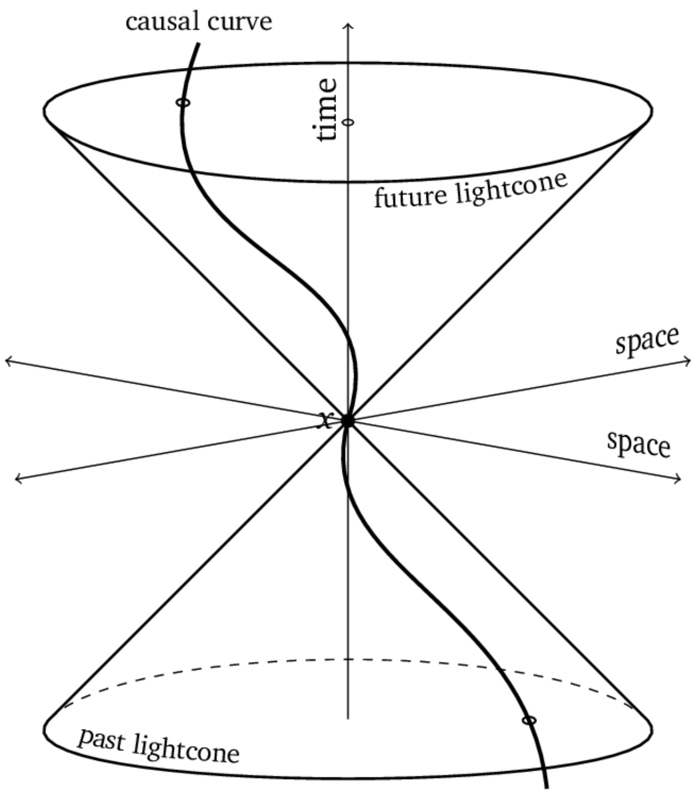
Dürr & Ben-Menahem (2022):
→ argue *all* of them are "unwarranted";
→ thus "short-circuiting" and "by-passing" the proof; "defeating its purpose".
→ instead adopt a selective anti-realism (not like Reichenbach's conventionalism!).

(CONF) Restriction to conformally equivalent spacetimes

At first sight, this seems the way the go: we should be allowed to look *everywhere* for an empirically equivalent geometry!

Yet, W&M present the restriction to conformal spacetimes (**CONF**) as a good thing:

> "*Note, though, that requiring conformal equivalence only strengthens our results. If the conventionalist cannot accommodate conformally equivalent metrics, then a fortiori one cannot accommodate arbitrary metrics; conversely, if Reichenbach's proposal fails even in the special case of conformally equivalent metrics, then it fails in the case of arguably greatest interest.*" (W&M, p. 237)



This is really two claims, I think:

1) For a given metric, the conformally equivalent metrics are most interesting.
2) If there is no F_{ab} for $\tilde{g}_{ab} = \Omega^2 g_{ab}$, then Reichenbach is a fortiori wrong.

But this slides between two different targets:
Does **Proposition 2** disprove a universality claim or an existence claim?

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Reichenbach's theorem θ : a universality claim

Theorem θ: "Given a geometry *G'* to which the measuring instruments conform, we can imagine a universal force *F* which affects the instruments in such a way that the actual geometry is an arbitrary geometry *G*, while the observed deviation from *G* is due to a universal deformation of the measuring instruments."[1]

Theorem is radical!

It is a **universality** claim: *Any* metric g can then reproduce the same empirical content as *any other* metric $\tilde{g}$

Theorem θ is an indirect target of W&M's theorem.

But the negation of a universality claim remains true under generalisation!

→ For theorem θ, there is no inconsistency proof!

Universality vs. existence: no loopholes to save theorem θ

- Negation of universality generalises: $(\forall g \forall \tilde{g}) \rightarrow (\forall g\ \forall_{conf} \tilde{g})$.
- W&M show $\neg(\forall g \exists_{conf} \tilde{g})$, which implies $\neg(\forall g \forall_{conf} \tilde{g})$.
- Thus (*modus tollens*): $\neg(\forall g \forall \tilde{g})$.

So theorem θ fails for relativistic spacetimes *without* (**CONF**).
(nor (**NORM**), (**RIEM**), (**TOPO**), …)
→ The only working posit "loophole" is (**FORCE**).

But this is **not** the claim modern conventionalists have in mind!

- (**ALT-ACC**): "Geometric alternatives for general-relativistic acceleration of a test-particle, $\xi^b \nabla_b \xi^a$ must *exist*".
- This targets Existence $(\forall g\ \exists \tilde{g})$, not Universality $(\forall g\ \forall \tilde{g})$.

Universality (UDT-$\forall g\ \forall \tilde{g}$):
Any metric can equally represent the empirical content as *any* other metric, if F_{ab} exists.

Restricted Universality (UDT-$\forall g\ \forall_{conf} \tilde{g}$): *Any* two conformally equivalent metrics can equally represent the empirical content, if F_{ab} exists.

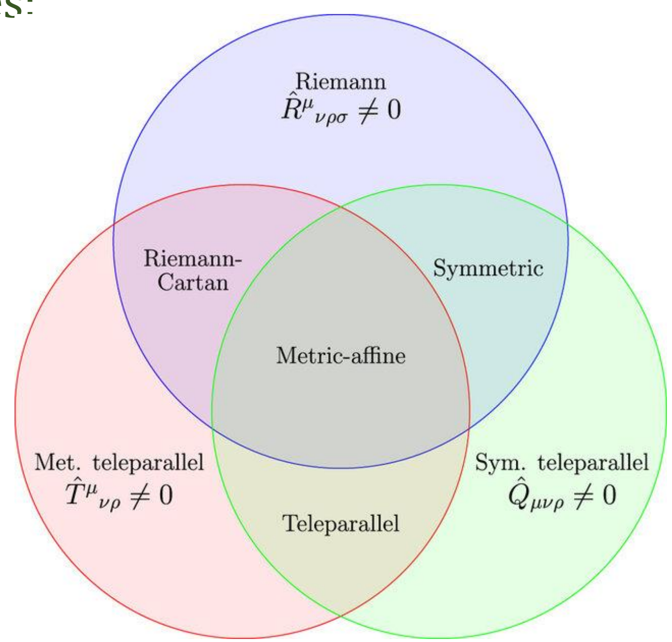
Restricted Existence (UDT-$\forall g\ \exists_{conf} \tilde{g}$):
For *any* metric, there *exists* another metric that can equally represent the empirical content, if F_{ab} exists.

Existence (UDT-$\forall g\ \exists \tilde{g}$):
For *any* metric, there *exists* another metric that can equally represent the empirical content, if F_{ab} exists.

¬(RIEM-SYMM): generalisation to torsionful spacetimes

There exist (initially) viable non-Riemannian theories: the geometric trinity of gravity.

Let's include teleparallel gravity by allowing for not-necessarily-symmetric spacetimes:



The torsionful relativistic case (Proposition 3.)

Let (M, g_{ab}) be a relativistic spacetime, let $\tilde{g}_{ab} = \Omega^2 g_{ab}$ be a metric non-trivially conformally equivalent to g_{ab} and let ∇ and $\tilde{\nabla}$ be not-necessarily-symmetric derivative operators compatible with g_{ab} and and $\tilde{g}_{ab}$, respectively. Then there is no tensor field F_{ab} such that an arbitrary curve γ is a geodesic relative to ∇ if and only if its acceleration relative to $\tilde{\nabla}$ is given by $F^a{}_n \tilde{\xi}^n$, where $\tilde{\xi}^n$ is the tangent field to γ with unit length relative to $\tilde{g}_{ab}$. (Mulder 2025a, p. 14)

Start from Jensen (2005): $C^a{}_{bc} = \frac{1}{2} g^{an}(\nabla_n g_{bc} - \nabla_b g_{nc} - \nabla_c g_{bn}) + K^a{}_{bc}$.

→ proof goes through in a trivially analogous manner, since the contorsion tensor $K^a{}_{bc}$ is independent of the metric.

Weatherall & Manchak's proof *generalises* under the rejection of (**RIEM-SYMM**).

¬(CONF)?: universality vs. existence


"*Note, though, that requiring conformal equivalence only strengthens our results. If the conventionalist cannot accommodate conformally equivalent metrics, then a fortiori one cannot accommodate arbitrary metrics*; conversely, *if Reichenbach's proposal fails even in the special case of conformally equivalent metrics, then it fails in the case of arguably greatest interest.*" (W&M, p. 237)

1. The first part is true if the target is Universality **($\forall g\ \forall \tilde{g}$)**, because (as before) ($\forall_{conf}\, g\ \forall_{conf}\, \tilde{g}$) → **($\forall g \forall \tilde{g}$)**.
   → but it is false if the target is Existence **($\forall g\ \exists \tilde{g}$).**

2. The second part takes the target Existence **($\forall g\ \exists \tilde{g}$)** *on the condition* that **(CONF).**

Reichenbach regarded causal statements as non-conventional, so he should (on this interpretation of Reichenbach) be committed (on his own terms) to the conformal part of the metric as non-conventional (Malament (1985)).

So requiring conformal equivalence does not always strengthen the result.
→ Rejecting the causal theory of time, Existence **($\forall g\ \exists \tilde{g}$)** may still hold for non-conformally equivalent spacetimes.

A programmatic attitude: the space of spacetimes

Rather than undermining the proof, like (D&BM, 2022), there is a more constructive attitude:

> Since the theorem contained in their Proposition 2 is both valid and non-trivial, we take there to be good cause to explore its implications as a `go theorem' in the context of the negation of the various physical, mathematical and framework assumptions. (Tasdan_& Thébault 2023, p.492)

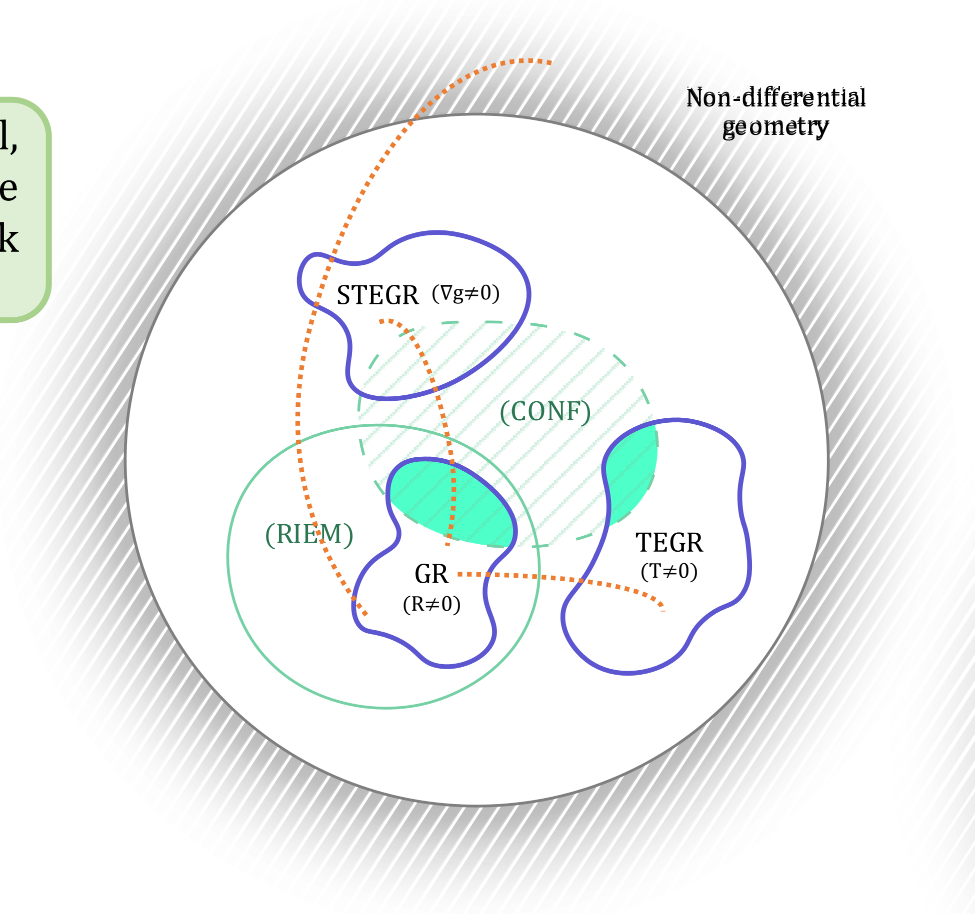
Then with a programmatic attitude, there's lots to do with the assumptions! → Both for classification and for heuristics.

¬(**RIEM**): using a general connection and disproving Existence ($\forall \boldsymbol{g}\ \exists \tilde{\boldsymbol{g}}$)

¬(**DIM4**): opens up a known way to Kaluza-Klein theory.

¬(**HAUS**): GR without differential geometry? → e.g., Einstein algebras.

Reichenbach's anomalies suggestion: {G+F+A, G'+F'+A' , G''+F''+A''}.



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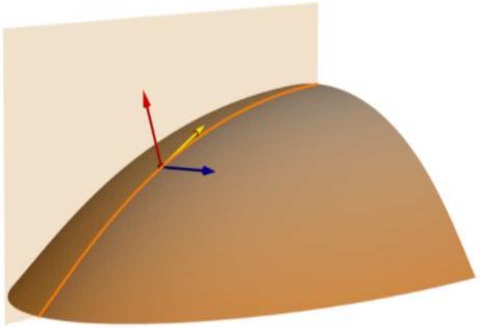
Effects and FORCEs



A universal force field (based on Weatherall & Manchak (2014)):

- (**FORCE-a**) some physical quantity acting on a massive body or point particle;
- (**FORCE-b**) represented a rank-2 tensor (field) F_{ab} ;
- (**FORCE-c**) the total force on a particle at a point must be proportional to its acceleration there.

> […] we believe that any reasonable account of "force" or "force field" in a Newtonian or relativistic framework would need to agree on at least this much, and so when we refer to forces/force fields "in the standard sense," we have in mind forces or force fields that have the character we describe here. (W&M, fn. 5, p. 235), p. 14)

[1] Generally the force F is a tensor. If $g'_{\mu\nu}$ are the metrical coefficients of the geometry G' and $g_{\mu\nu}$ those of G, the potentials $F_{\mu\nu}$ of the force F are given by

$$g'_{\mu\nu} + F_{\mu\nu} = g_{\mu\nu} \quad \mu\nu = 1, 2, 3$$

The measuring rods furnish directly the $g'_{\mu\nu}$; the $F_{\mu\nu}$ are the "correction factors" by which the $g'_{\mu\nu}$ are corrected so that $g_{\mu\nu}$ results.

Dürr & Ben-Menahem against (**FORCE**)

D&BM say (**FORCE**) is overly restrictive because it is **too conservative**.

Why not consider a broader notion of ‘interaction’, including **rank-3 tensors** $\boldsymbol{F^{a}{}_{bc}}$?

1) Historically, the force concept is variable (cf. Jammer 1957, Hesse 1961):

> *“to ennoble conservativeness as an unqualified virtue per se, we regard as unduly reactionary [...]; such a view would be amply belied by the history of science.” (D&BM 2022, p. 163).*

2) Nor can one appeal to simplicity of tensor rank as a criterion for theory choice:

> *Simplicity “is a pragmatic maxim par excellence (see e.g. Bunge, 1963). Its connection with truth is evidently fragile. It has its place as a heuristic rule of thumb in theory construction and assessment in praxi: it recommends to first inspect conservative hypotheses, which fit in most easily with our background assumptions, before considering more radical ideas.” (D&BM, p. 163)*

Comparative function: No “(**FORCE**)-equivalent” to GR like NCT/NG

At least (**FORCE**) has a *comparative* function.

Is there a *Newtonian* force theory in flat space, equivalent to GR?

Newtonian gravitational forces are like universal forces (Glymour; Dieks 1987), because they
(a) affect all materials in the same way;
(b) cannot be screened-off by insulating walls.

Newton-Cartan theory has causally efficacious space and time …
… and is equivalent to Newtonian gravity, which is a (**FORCE**)-theory.

No such “(**FORCE**)-equivalent” exists for to GR” analogous to Newtonian gravity

	Geometric theory	FORCE theory
Non-relativistic	Newton-Cartan theory	Newtonian gravity
Relativistic	General relativity	

Tensor rank simplicity

“at some abstract level of description, a thesis [i.e. conventionalism] is irrefutable. But at that same level of abstractness, as has often been observed, it is also ***uninteresting****. We can be conventionalists about geometry, perhaps, but in the same way that* ***we could be conventionalists about anything****.”“ (W&M, p. 234)*

… if one is willing to postulate enough—an evil demon, say.” (W&M, p. 246)

“Reichenbachian conventionalists would squarely protest: the persuasiveness of conventionalism doesn't depend on how ***cumbersome*** *geometrically alternative descriptions are* ***but on their very existence****. W&M repudiate the alternative conventions* ***on the grounds of simplicity****. This, however, cuts no ice against conventionalism of any form: conventionalists expressly concede that geometrically alternative accounts may be less simple..” (D&BM, p. 161)*

Perhaps W&M rely on something like this?:

Criterion of Tensor Rank Simplicity*. When multiple empirically equivalent theories (or models) differ with respect to the order of ranks of the tensors they employ, then (absent salient other differences) it is epistemically virtuous to prefer the theory (or model) that employs tensors of the lowest rank.* (Mulder 2025b, p.8)

The Demon Tensor

But this does not seem to me what is going on!
→ They do consider rank-3 tensor fields
→ They reject them for being trivial (two connections can always be connected by such a field)
→ No epistemically questionable "Criterion of Tensor Rank Simplicity"

D&BM really seem to suggest to suspend on judgement on the basis of the possibility of a "Demon Tensor":

The Demon Tensor. A tensor $F_{abcd\ldots n}$ of high but unspecified rank n, which encodes universal effects. Given the mathematical leeway in the form of unspecified degrees of freedom of such a tensor, one can potentially accommodate any dynamical structure, the abstractness of which raises worries of Cartesian "evil demon" skepticism. (Mulder 2025b, p.8)

“Interesting” and “non-trivial” alternatives: an olive branch?

Both sides in this debate search for a conceptual middle ground about the force concept

“at some abstract level of description, a thesis [i.e. conventionalism] is irrefutable. But at that same level of abstractness, as has often been observed, it is also ***uninteresting****. We can be conventionalists about geometry, perhaps, but in the same way that* ***we could be conventionalists about anything****.”“ (W&M, p. 234)*

D&BM reject of *trivial* semantic holism:

*“****Reasonable*** *alternatives to these assumptions exist that open up geometric alternatives [...]*
trivial semantic conventionalism*—trafficking in the conventionality of merely representational/linguistic differences of synonymous/logically equivalent content—is of little relevance to the debate.” (D&BM, p. 170)*

→ this is why Reichenbach is “not entirely wrong” but still wrong.

Both sides can be seen as reacting to underdetermination via *le bon sens géométrique*!

Le bon sens (géométrique)

Bon sens is a **supra-empirical response** to underdetermination induced by holism.
→ Based on expert *judgement*: neither logic not empirical investigation can jully justify a choice:

> For it may be that good sense permits us to decide between two physicists. [...] But these reasons of good sense do not impose themselves with the same implacable rigor that the prescriptions of logic do. There is something vague and uncertain about them; they do not reveal themselves at the same time with the same degree of clarity to all minds. Hence, the possibility of lengthy quarrels [...]. (Duhem 1906, p. 217)

For our context:

> ***Le bon sens géométrique***. On the Helmholtzian assumption that there are geometrical facts, one needs to appeal to a form of Duhemian good sense in order to state them. This judgement goes beyond purely logical and empirical considerations and is based on expertise and understanding of differential geometry, through careful weighing of the costs and benefits of adding or alleviating constraints on the dynamics and the associated physical characterisation of that dynamics. Trivial semantic holism and this geometric judgement are two sides of the same coin. (Mulder 2025b, p. 13)

However, without a clear reformulation of GR in terms of rank-3 tensor fields, D&BM's proposal is **very difficult to hold onto in good sense**. There is no need for a lengthy quarrel!
→ For example: one must adjust the law of conservation of energy.

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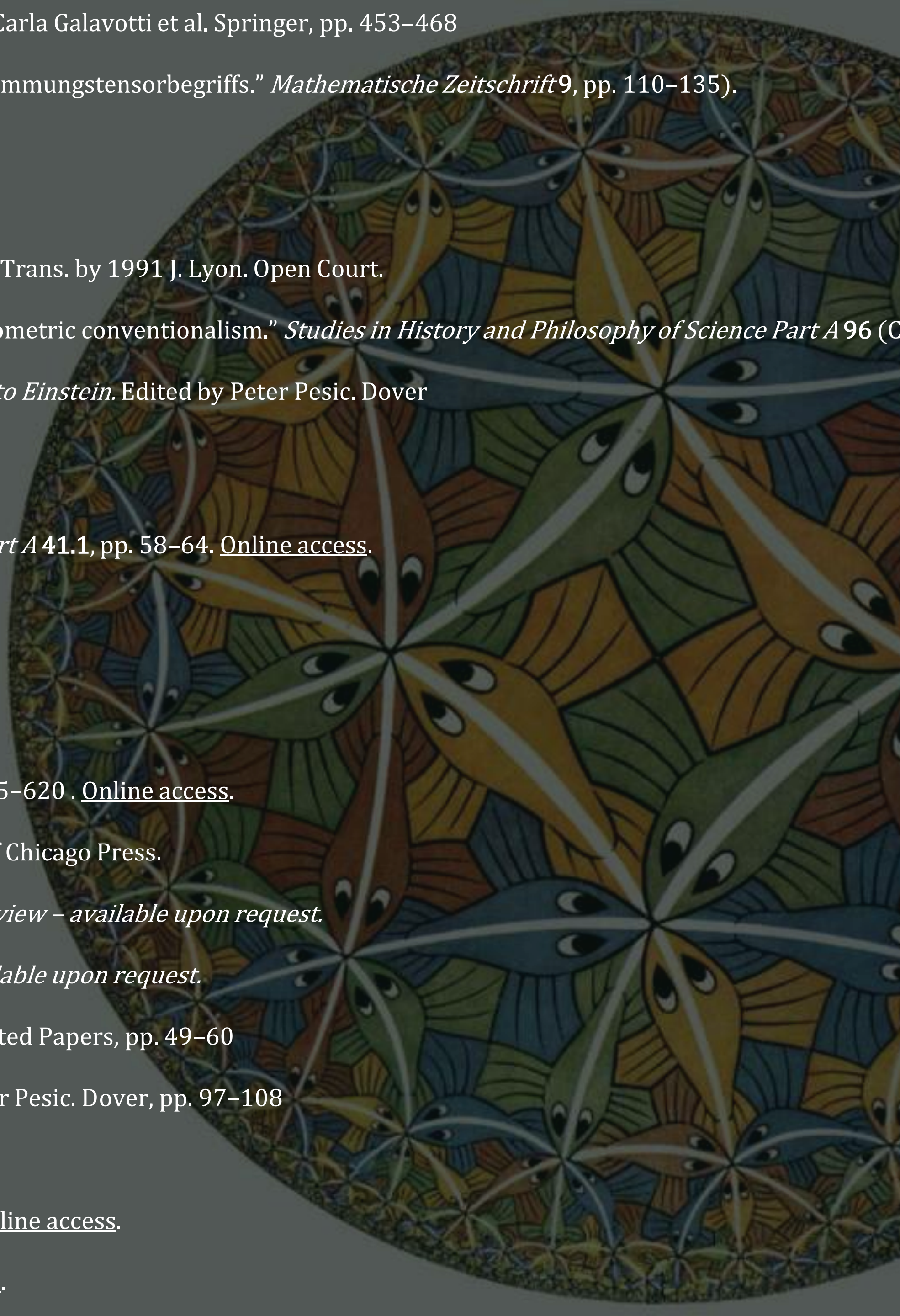

¬(CONF): strengthening Proposition 2

Recently, Bryan Roberts (2025) strengthened the proof to conformally inequivalent spacetimes to a given metric:

Start with acceleration eq.: $\xi^b \tilde{\nabla}_b \xi^a = \xi^b \nabla_b \xi^a + C^a_{\ bc}\xi^b\xi^c$.

For any ∇geodesic ξ^a, we have: $F^a_{\ b}\xi^b = mC^a_{\ bc}\xi^b\xi^c$.

Take three timelike geodesics: χ^a, ψ^a and $\chi^a + \psi^a \neq 0$ at p.
Each satisfy the force law at point p:

$$F^a_{\ b}\chi^b + F^a_{\ b}\psi^b = F^a_{\ b}(\chi^b + \psi^b)$$
$$mC^a_{\ bc}(\chi^b\chi^c + \psi^b\psi^c) = mC^a_{\ bc}(\chi^b + \psi^b)(\chi^c + \psi^c)$$
$$0 = C^a_{\ bc}(\chi^b\psi^c + \chi^c\psi^b)$$
$$\rightarrow C^a_{\ bc} = 0 \text{ (since } C^a_{\ bc} \text{ is symmetric)}$$

→ For disproving Universality, (**CONF**) was already not needed.
→ For disproving Existence, (**CONF**) *is* a loophole, but this seems to be now plugged by Roberts. (Nor need for (**NORM**) !)
→ But only partially: without (**NORM**) there are still alternative metrics!

<u>The conformally inequivalent case (Proposition 4.)</u>

Let (M, g_{ab}) and $(M, \tilde{g}_{ab})$ be relativistic spacetimes, with respective Levi-Civita connections ∇ and $\tilde{\nabla}$. Suppose there is some tensor field F_{ab} such that, whenever ξ^a is a timelike ∇-geodesic, $F^a := \xi^a F^a_{\ b}\xi^b$ satisfies Newton's equation with respect to $\tilde{\nabla}$. That is, $F^a = m\xi^b\tilde{\nabla}_b\xi^a$ for $m > 0$. Then $\nabla = \tilde{\nabla}$ and $F^a_{\ b} = 0$. (Roberts, 2025, p. 22)