The open-textured (history of the) “force” concept in modern relativistic geometric conventionalism

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Universal effects and universal forces: 100 years of disagreement

Scientific goals and the history of the open-textured concept of “force”

Conceptual goals: fine-graining and coarse-graining and Reichenbach’s hyperrealism

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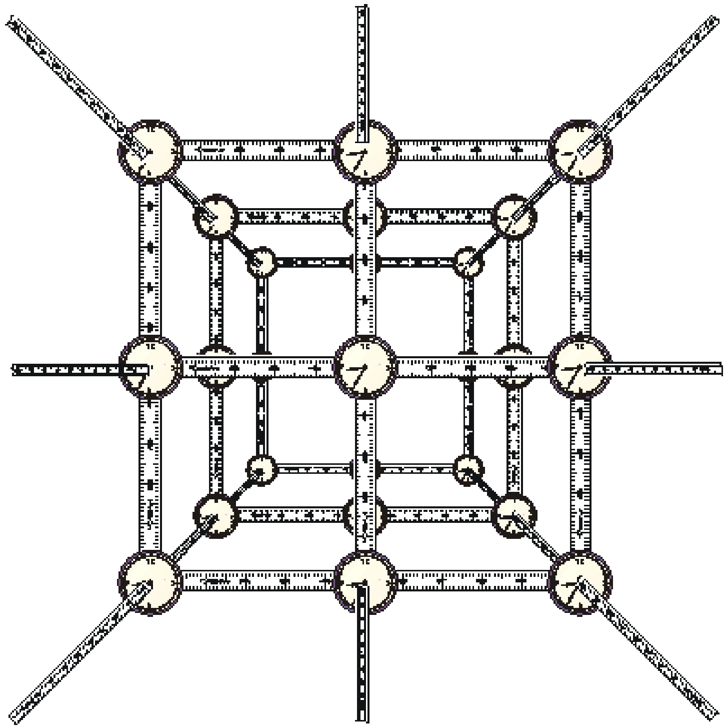
Universal “forces”: 100 years of trading off geometries (1/3)

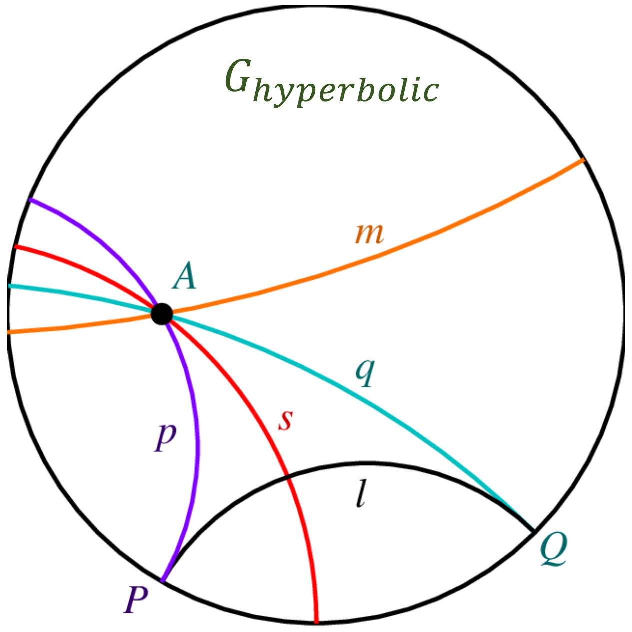
Physical geometry (Helmholtz 1866): empirical question for the *physical* geometry of the world: rods and clocks.



Universal “forces”: 100 years of trading off geometries (2/3)

Physical geometry (Helmholtz 1866): empirical question for the *physical geometry* of the world: rods and clocks.

Geometric holism based on Poincaré’s equivalent proofs (1891): there are many empirically equivalent combinations of geometries [G] and “universal forces” (“universal effects”) [F]: {G+F , G’+F’ , G’’+F’’, ... }.



Universal “forces”: 100 years of trading off geometries (3/3)

Physical geometry (Helmholtz 1866): empirical question for the *physical geometry* of the world: rods and clocks.

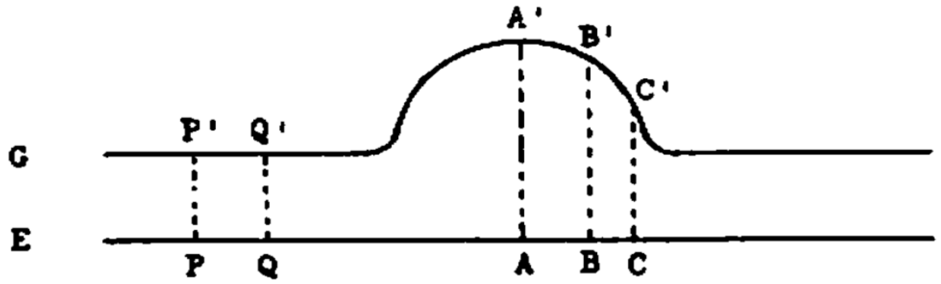
Geometric holism based on Poincaré’s equivalent proofs (1891): there are many empirically equivalent combinations of geometries [G] and “universal forces” (“universal effects”) [F]: {G+F , G’+F’ , G’’+F’’, ... }.

Conventionalism about space(time): ascertaining the physical geometry requires (in some way) a conventional choice.
→ Loosely associated with Poincaré, Duhem, Schlick, Carnap, and others, and Reichenbach (1926, Sec. 8).:

Theorem θ: “Given a geometry *G’* to which the measuring instruments conform, we can imagine a universal force *F* which affects the instruments in such a way that the actual geometry is an arbitrary geometry *G*, while the observed deviation from *G* is due to a universal deformation of the measuring instruments.”[1]

Theorem θ shows all geometries to be equivalent; it formulates the *principle of the relativity of geometry*. It follows that it is meaningless to speak about one geometry as the *true* geometry.

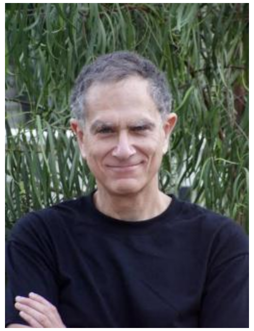
An effect (force) is **universal** iff it
1. cannot be screened off by insulating walls
2. acts equally on all materials/particle species
Otherwise it is a **differential** effect (or force)



Disagreement is absolute!







physical meaning of “force” is much more substantive

“misleading”

Not a force in the “standard sense”

“Funny force”

“yet another skeptical fantasy”

“fairies at the bottom of my garden”

No epistemological objection can be made against the correctness of theorem θ.

You should interpret ‘force’ “metaphorically”

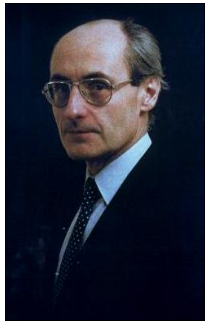
Better to say: “universal interactions”

Better to say: “universal effects”

Has good applications in physical practice

“Trivially correct”








Substantive dispute: scientific vs. conceptual goals

Assumptions about rigid measuring rods serve as "preconditions both of the individuation of physical magnitudes and of their measurement, and, as such, they are necessary to approach the world in the first instance" (Padovani, p.49).

Phil. goal: regulating the **concept of 'force'** as '**change of physical geometry**'.
→ Linked definition of physical geometry and universal effects (**UE**).
→ Coordinative definitions operationally link the metric tensor to the empirical world via rigid bodies: the reality of (**UE**) is a matter of convention.
→ Allows **measurement** of physical geometry as an empirical quantity.

"But this answer only shifts the problem. The usual physical meaning of "force" is much more substantial than a mere stipulation about the presence or absence of geometrical changes. Reichenbach's conventional definition of force is quite debatable. (Acuña, p.463)

Scientific goals as putting **constraints** on universal forces (**UF**):

- Spinning particles: $F_{ab}\,\xi^a = R_{abcd}S^{cd}\xi^a$.
- Gyroscopic drift: $F_{ab}\,\xi^a = H_{ab}\xi^a$.
- Gravitational waves for geodesic deviation: $g^{ab} = \eta^{ab} + h^{ab}$ for h^{ab} *small.*

Padovani (2017). "Coordination and Measurement."
Acuña (2013). "Artificial Examples of Empirical Equivalence".

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Forces and FORCEs


Weatherall & Manchak (2014) formulate a constraint as a "standard force field":

(**FORCE-a**) some physical quantity acting on a massive body or point particle;
(**FORCE-b**) represented a rank-2 tensor (field) F_{ab} ;
(**FORCE-c**) the total force on a particle at a point must be proportional to its acceleration.

Like in the Newtonian framework: $a^a = \tilde{\xi}^b \tilde{\nabla}_b \tilde{\xi}^a + F_b{}^a \tilde{\xi}^b$.

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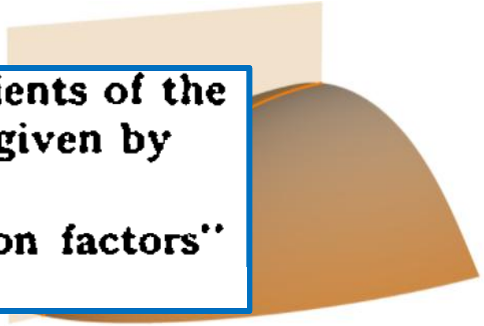
[1] Generally the force F is a tensor. If $g'_{\mu\nu}$ are the metrical coefficients of the geometry G' and $g_{\mu\nu}$ those of G, the potentials $F_{\mu\nu}$ of the force F are given by

$$g'_{\mu\nu} + F_{\mu\nu} = g_{\mu\nu} \quad \mu\nu = 1, 2, 3$$

The measuring rods furnish directly the $g'_{\mu\nu}$; the $F_{\mu\nu}$ are the "correction factors" by which the $g'_{\mu\nu}$ are corrected so that $g_{\mu\nu}$ results.



Let us call this the **universal standard force** (**USF**).
→ (**FORCE-a** and **FORCE-c**): pulling you off a geodesic.
→ (**FORCE-b**) now also represented by a 2-tensor.

Weatherall & Manchak: "the geometry of conventionality" in GR

Given (**FORCE**), can these trade-offs *always* be made given the mathematics of modern spacetime theories.

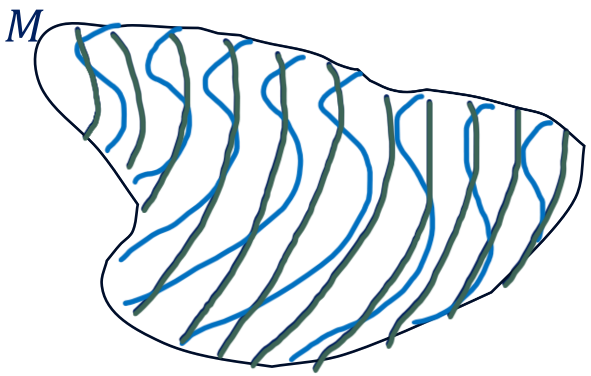
Weatherall & Manchak prove:

<u>Newtonian gravity</u>: yes, this is always possible.
<u>General relativity</u>: no, this is not always possible.

Striking:
(i) Much-needed rigour to a conceptual debate;
(ii) Explicit theory-dependence.

But a rather unconventional anti-conventionality!
(iii) No engagement with Reichenbach's goals;
(iv) How good are the assumptions? In particular (**FORCE**).

Spacetime (M, g, ∇)
with geodesics: $\xi^b \nabla_b \xi^a = 0$



Spacetime $(M, \tilde{g}, \tilde{\nabla})$
with geodesics: $\tilde{\xi}^b \tilde{\nabla}_b \tilde{\xi}^a = 0$

Spacetime $(M, \tilde{g}, \tilde{\nabla})$
with dynamics: $\tilde{\xi}^b \tilde{\nabla}_b \tilde{\xi}^a + (\mathbf{UF}) = 0$

Dürr & Ben-Menahem against (FORCE)


Dürr & Ben-Menahem (D&BM) say (**FORCE**) is **overly restrictive** because it is conservative.
→ Why not consider a broader notion of 'interaction', including rank-3 tensors ${F^a}_{bc}$?

1) One cannot appeal to simplicity of tensor rank as a criterion for theory choice:

> Simplicity "is a pragmatic maxim par excellence (see e.g. Bunge, 1963). Its connection with truth is evidently fragile. It has its place as a heuristic rule of thumb in theory construction and assessment in praxi: it recommends to first inspect conservative hypotheses, which fit in most easily with our background assumptions, before considering more radical ideas." (D&BM, p. 163)

2) The force concept has historically proven to be considerably variable:

> "to ennoble conservativeness as an unqualified virtue per se, we regard as unduly reactionary [...]; such a view would be amply belied by the history of science."
>
> "[...] terms, such as "force", can – occasionally quite radically – change their meaning. For the notion of "force" in particular such changes are well-documented (see e.g. Jammer,2011). Conventionalists, pace W&M's suggestion […], aren't committed to rejection (let alone denial of the existence) of such changes!." (Dürr & Ben-Menahem. p. 163).

Hesse (1959) and Jammer (1957) on the history of “force”

Two grand histories of the force concept.

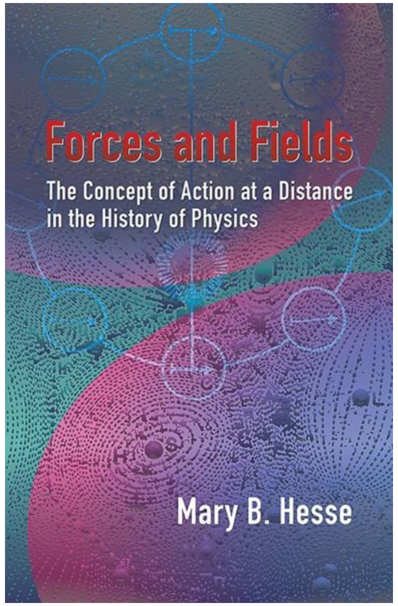
Both see progress as successive hair-splitting and conceptual enrichment:

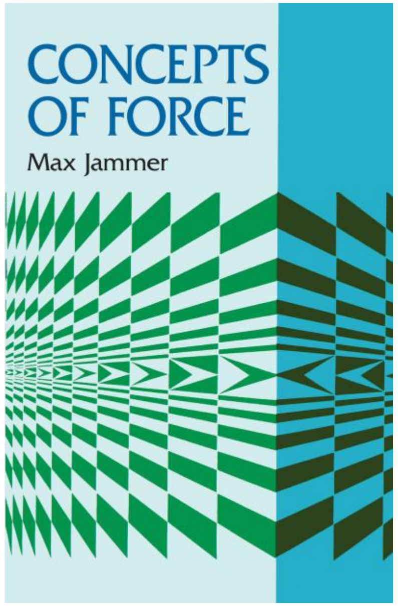
- roots is ancient **animistic** “tendency” or “striving”;
- “emancipated” (MJ, Ch.2) from **spiritual into natural** (e.g. Aristotle);
- attaining **quantification** with Kepler;
- distinction between **forced and force-free** motion (Galileo);
- distinction whether forces can be **screened off** or not with Huygens;
- connoted with **causes and causality** with Kant;
- by 1950 become purely relation (MJ, Ch.12, challenged by MH).

Jammer and Hesse disagree on the **concept of “physical concept”** and corresponding historical methodology.

Jammer. concepts are **analytically** individuated.

Hesse. *theoretical models are always richer than the phenomena they aim to explain*: empirical concepts are not nailed down: **open texture**.





Open-texture vs. analyticity


Jammer:
Physical concepts are specified by necessary and sufficient conditions.
Vagueness belongs to the context of discovery until concepts find an "exact definition in science" (p.2).
Progress = supplanting one analytic concept with a new one.

"from the standpoint of the history of ideas, the most interesting and important part of its biography is passed [...]." (Jammer, p.2)

Hesse:
Scientific concepts are open-textured: they have an open-ended fringe of meaning, i.e. vagueness.
This allows concepts to be transplanted from one context into the next.
→ Progress in science is fueled by such transplantations

Back to universal forces:

- D&BM do not want Reichenbach's (**UE**) to be unconstrained, as Reichenbach did.
- But they do not want **permanent** constraints on (**UF**): there are no necessary conditions.
  → in line with open-texture: all conditions should be upendable!

- W&M's (**FORCE-a**)-(**FORCE-c**) are a partial closure of "force": necessary but not sufficient.
  → Significant upshot: mathematical traction.

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Conceptual goals: fine-graining and coarse-graining and Reichenbach’s hyperrealism

Engineering fine concepts

Let's conceptualise a concept as a simple **list of defining clauses:**

- Adding clauses is called conceptual **fine-graining**, making it logically stronger.
- Deleting clauses is conceptual **coarse-graining** (weaker)

Refining tools of thought:

- Sufficiently flexibly to accommodate **change** *and* **similarity**
- Sufficiently general to adapt to different meaning-assignments (e.g. Carnap, Dewey, cf. Westerblad 2024)
- Merely **semantic**: endorsing these concepts is an epistemic question.

Try to break $g_{ab} - F_{ab} = \tilde{g}_{ab}$ underdetermination via:

- <u>fine-graining 'F' for scientific goals</u>:
  - W&M's (**FORCE-a**)-(**FORCE-c**) are added defining clauses to (**UE**), turning them into (**USF**)
  - then disallow empirically-equivalent formal constructions that do not meet those constraints

- <u>coarse-graining 'G' for philosophical goals</u>:
  → identifying $g_{ab} - F_{ab}$ and $\tilde{g}_{ab}$ as equally capable of representing a weaker concept of physical geometry G.

A suggestion: coarse-graining “physical geometry”

> Since there is no demonstrable difference produced by universal [effects], the conception that the transported measuring rod is deformed by such forces can always be defended. No object is rigid relative to universal forces. (p. 22)

- A Universally Rigid Body measures **Universal Physical Geometry** (**UPG**)
  → Picking a Universally Rigid Body is beyond our capacities.

I agree epistemically but disagree semantically:

- But Reichenbach’s (**UPG**) is too fine-grained!
  → metric is taken to *essentially* represent geometry.
- Better: no conceptual difference between being ‘**deformed by universal forces**’ and ‘**having distinct universal physical geometry**’.

A Differentially Rigid Body measures **Differential Physical Geometry** (**DPG**).
→ but we only have empirical access to the coarse-grained (**DPG**)
→ differences in (**UPG**) should be seen as having the same capacity to represent (**DPG**).

Universally Rigid Body. A solid material object that does not deform while being transported, whether due to differential forces or universal effects.

Differentially Rigid Body. A solid material object that does not deform under the influence of differential forces while being transported, either by not being affected by these forces or if their effects are cancelled out. They may deform under universal effects.

Coarse-graining as a general response to underdetermination?

What does it mean to take a mathematical formalism literally?

I propose a "Semantic Grain Framework":

Mathematical side: picking out / selecting a number of mathematical items (or amount of structure)
→ coarse-graining is picking out fewer items.

Conceptual side: engineering concepts as lists of defining clauses,
→ coarse-graining is deleting some of these clauses.

'Semantic equilibration': matching the maths and the concepts satisfactorily, with neither coming first.

Semantic coarse-graining:

Pick out not all but just a portion of the formalism. Simultaneously, weaken two or more hitherto distinct concepts such that they collapse into a single concept, in a way that the weaker concept correlates with this smaller portion of the formalism. Semantic coarse-graining is not always possible.

"Science aims to give us, in its theories, a literally true story of what the world is like; and acceptance of a scientific theory involves the belief that it is true." (Van Fraassen 1980, p. 8)

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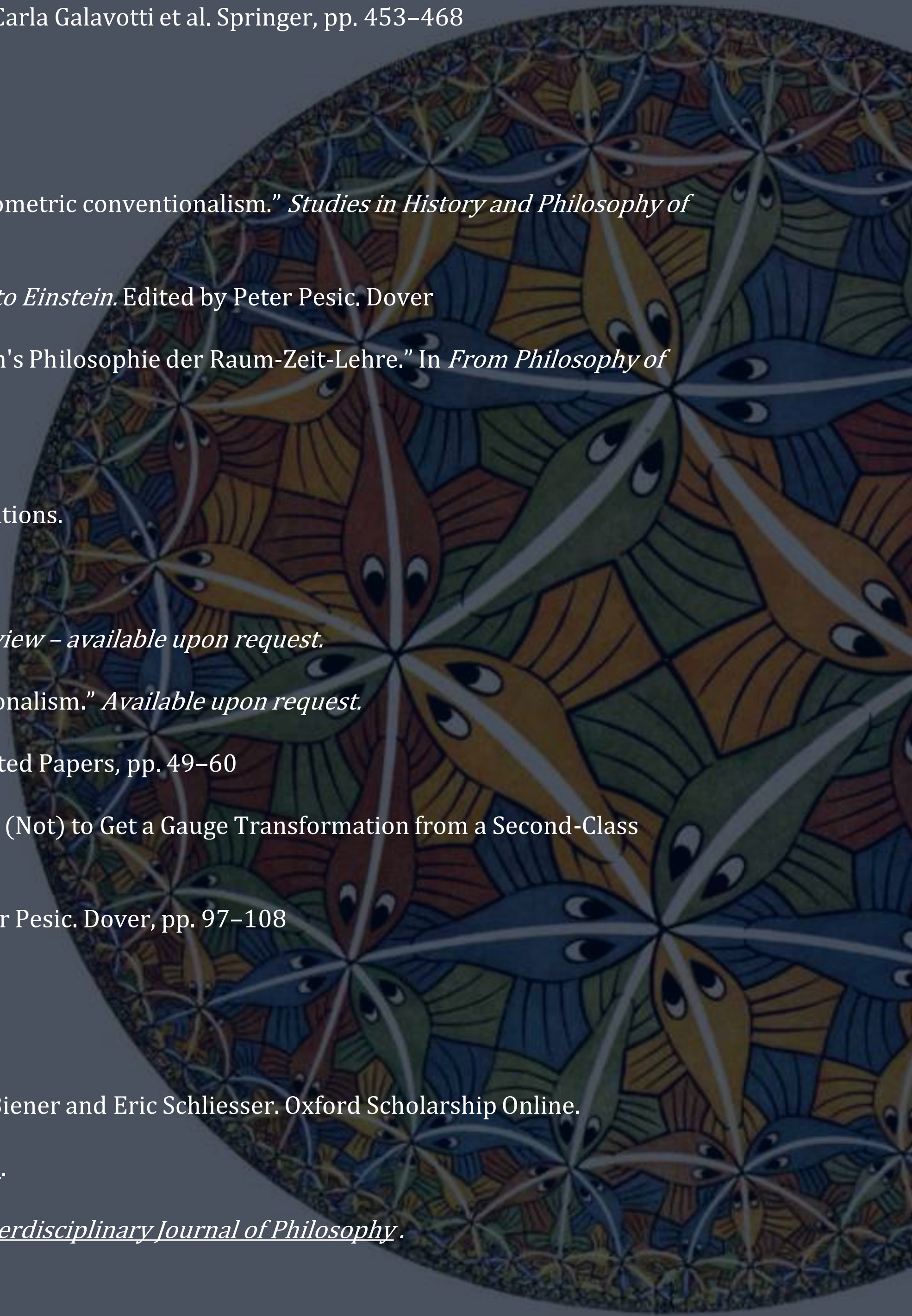

Universal effects:
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Conceptual goals: fine-graining and coarse-graining

Are universal forces and physical geometry equivalent?

Are UE and UPG theoretically equivalent (within GR)?

$F_{ab} \coloneqq g_{ab} - g'_{ab}$.

1. Take GR.
2. Select the set of all smooth metrics on the same manifold M, i.e. $G = \{g_1^{ab}, g_2^{ab}, g_3^{ab}, \ldots\}$
   → Point-wise substractions make sense
   → The difference of two symmetric (0,2) tensor fields is again a symmetric (0,2) tensor field.
3. Since $F_{ab} \coloneqq g_{ab} - g'_{ab}$, take $F = \{F_{(12)}^{ab} = g_1^{ab} - g_2^{ab}, F_{(13)}^{ab} = g_1^{ab} - g_3^{ab}, F_{(23)}^{ab} = g_2^{ab} - g_3^{ab}, \ldots\}$
4. So each element of F is a smooth section of the bundle S^2T^*M

If G has cardinality κ, then:

- the number of unordered pairs is at most $\binom{\kappa}{2}$
- But there may be that different pairs give the same $F_{(ij)}^{ab}$.
- So $\binom{\kappa}{2} \leq |F| \leq \kappa^2$.
- But the infinite case: $\kappa \leq |F| \leq \kappa^2$.

If you are merely defining F set-theoretically as the set of pairwise differences of metrics in G defined on M, then no additional structure is required to use universal effects.

→ These all give equivalent ways to measure Differentiated Physical Geometry

→ Reichenbach's aim to measure Universal Physical Geometry is too ambitious.

Reichenbach’s Universal Physical Geometry is too fine-grained

Reichenbach considers whether we can hold on to Euclidean geoemetry G_0.

Reichenbach’s solution proposes a methodological rule:

1. set $F = 0$ *by definition*.
2. Just use them on the assumption of the congruence of transported rods.
3. Correct for differential forces only.
4. Specific models of (**UPG**) become “true by convention”, relative to $F = 0$.

> One can “no more say that Einstein’s geometry is truer” than Euclidean geometry, than that we can say that the meter is a “truer” unit of length than the yard.” (1928, p.35)

→ That does not mean we’ve learned *nothing* non-conventionally true! (Reichenbach 1951, pp.136-7)

However:

- Reichenbah is maximally realist (indeed essentialist) about the metric tensor in isolation representing (**UPG**)
- but this is *more* than we have empirical access to: (**DPG**)
- Thus Reichenbach’s attempt to measure (**UPG**) is aiming for a too fine-grained concept

	Physical geometry	**Universal effects**
Ia	Euclidean	yes
Ib	non-Euclidean	no
IIa	Euclidean	no
IIb	non-Euclidean	yes

Reichenbach (1928). Raum und Zeit.
Dieks (1987). “Gravitation as a universal force.“
Reichenbach (1951). *The Rise of Scientific Philosophy*.

Evidential reasons for discarding UE?

"Gorce" and "Morce": $F = F_{gorce} + F_{morce}$

"I should tell him something like this. His theory is merely an extension of Newton's. If he admits that an algebraic combination of quantities is a quantity, then his theory is committed to the existence of a quantity, the sum of gorce and morce, which has all of the features of Newtonian force, and for which there is exactly the evidence there is for Newtonian forces. But *in addition* his theory claims that this quantity is the sum of two distinct quantities, gorce and morce. However, *there is no evidence at all for this additional hypothesis*, and Newton's theory is therefore to be preferred. (Glymour 1977, p. 237-238, *my emphasis*)

But I'm not sure: is (**UE**) really an *additional* hypothesis?

For $F_{ab} \coloneqq g_{ab} - \tilde{g}_{ab}$, we cannot see g_{ab} or $\tilde{g}_{ab}$ as "smetrics"
→ They are metrics!

Can we see as a F_{ab} as a "smorce"?

Glymour (1977). "The epistemology of geometry."
Smith (2014). "Closing the Loop: Testing Newtonian Gravity, Then and Now."
Ben-Menahem (2006). *Conventionalism*.

Trivial semantic holism: the OG of de-Ockamizations


Dürr & Ben-Menahem (2014) identify Reichenbach's trivial semantic holism:

- One can always split up any quantity into many terms and insist on one of them being "fundamental".
- For example the metric tensor: $\mathrm{g} = \boldsymbol{g}_1 + \boldsymbol{g}_2 + \boldsymbol{g}_3 + \boldsymbol{g}_4$.

But this is too easy: the $\boldsymbol{g}_i$ are meaningless!
→ $\mathrm{F}_{\mathrm{ab}} \coloneqq \mathrm{g}_{\mathrm{ab}} - \tilde{\mathrm{g}}_{\mathrm{ab}}$ is different: both g_{ab} and $\tilde{\mathrm{g}}_{\mathrm{ab}}$ are already metric tensors.

Brian Pitts (2022) coins the term de-Ockhamization:

- using more when less suffices by splitting one quantity into the sum of two is trivial (not false!).
- creating additional gauge freedoms
- for example: adding a constant to the electrostatic potential and insisting it is real!

→ Reichenbach's proposal is more like that.
→ But there are common solutions for that: equivalence.

Brian Pitts (2022). "First-Order Constraints, Gauge Transformations, De-Ockhamization, and Triviality"
Clark Glymour (1977). "The epistemology of geometry."
Dürr & Ben-Menahem (2022). "Why Reichenbach wasn't entirely wrong, and Poincaré was almost right [...]"