Metric degeneracy: why relativistic gravity is less susceptible to underdetermination and separatism than Newton-Cartan theory

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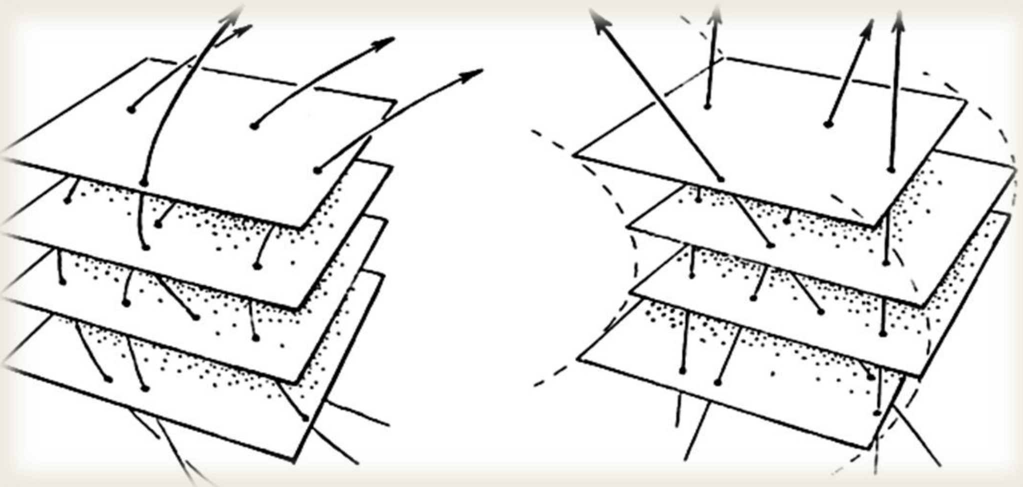

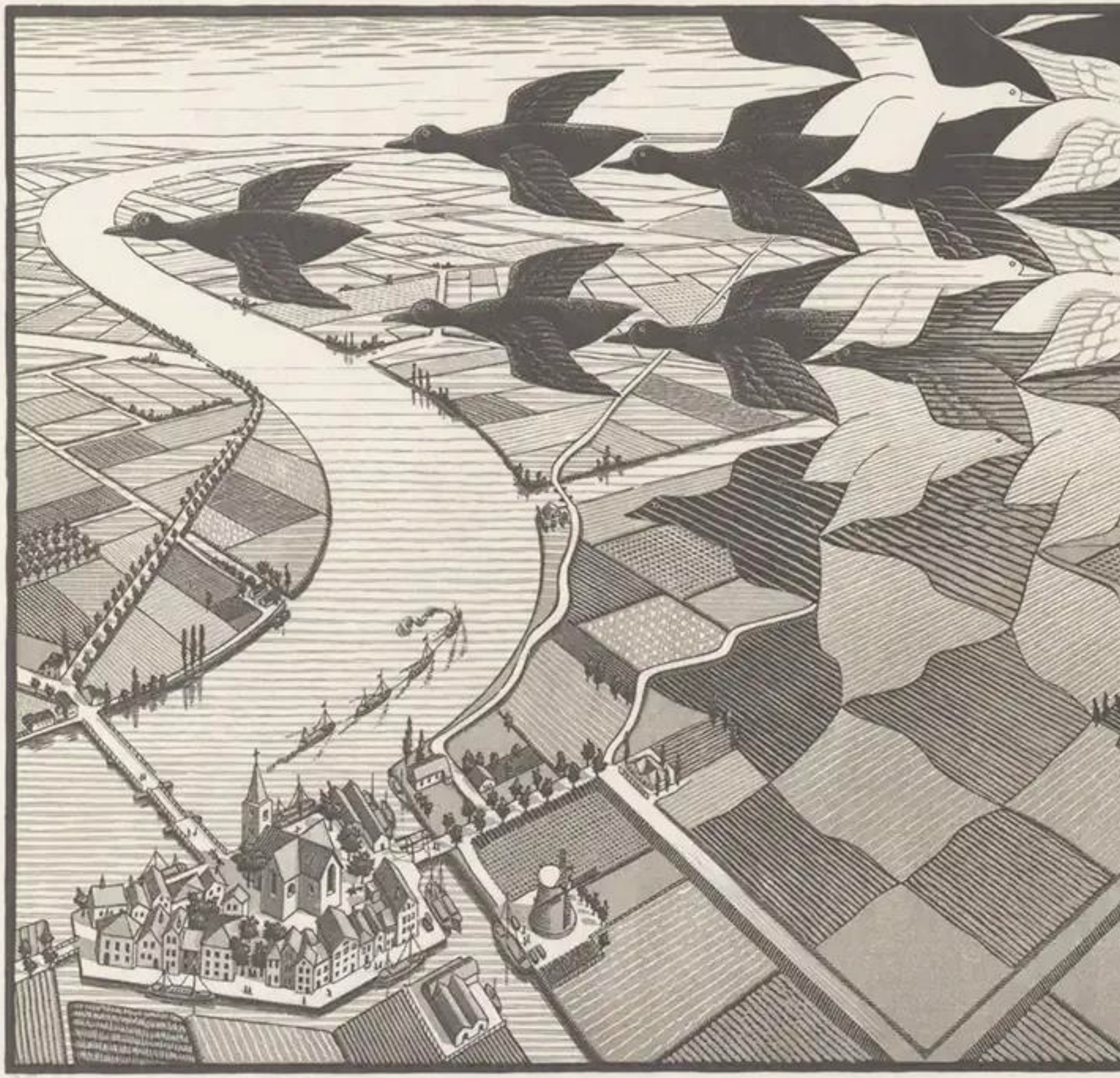

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Synopsis

1) Two proofs (Weatherall & Manchak's 2014) show that: non-relativistic gravity is more susceptible to conventionalism than general relativity.

→ Not clear why there is a discrepancy between the relativistic and the non-relativistic cases.

2) A recent metaphysical position about spacetime called "unitism" and its anti-thesis "separatism" (Gilmore, Costa, Calosi 2016):

→ Missing mathematical underpinnings for unitism / separatism.

Claim:
Metric degeneracy is a crucial disanalogy between relativistic and non-relativistic spacetime theories that underlies underdetermination and support for separatism.

Geometric conventionalism through universal effects

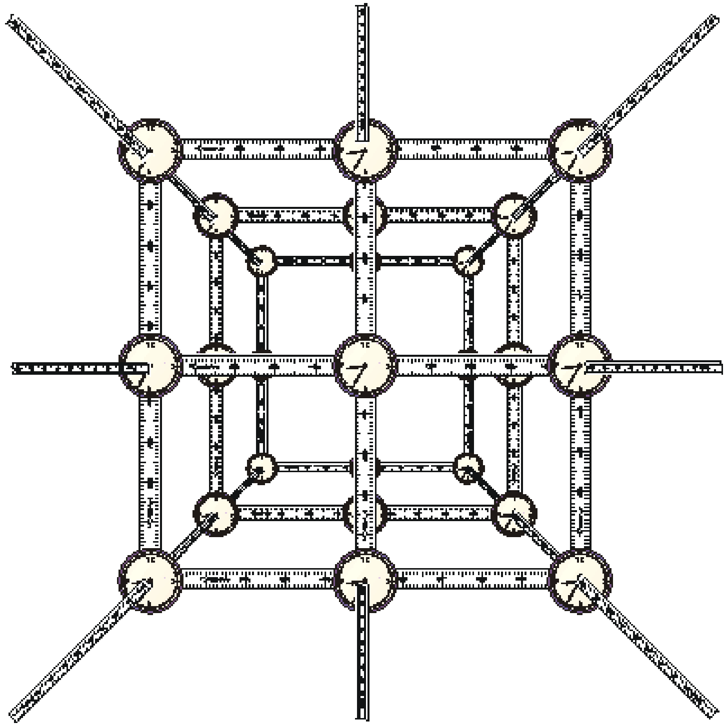
Universal forces and (non-) relativistic gravity theories

Metric degeneracy and Newton-Cartan theory

Unity, unitism, unification

Universal "forces": 100 years of trading off geometries (1/3)

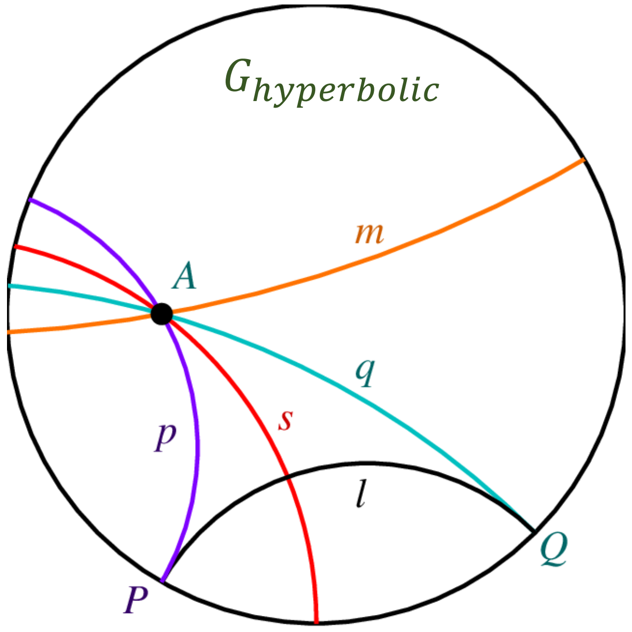
Physical geometry (Helmholtz 1866): empirical question for the *physical* geometry of the world: rods and clocks.



Universal "forces": 100 years of trading off geometries (2/3)

Physical geometry (Helmholtz 1866): empirical question for the *physical geometry* of the world: rods and clocks.

Geometric holism based on Poincaré's equivalent proofs (1891): there are many empirically equivalent combinations of geometries [G] and "universal forces" ("universal effects") [F]: {G+F , G'+F' , G''+F'', ... }.



Universal “forces”: 100 years of trading off geometries (3/3)

Physical geometry (Helmholtz 1866): empirical question for the *physical geometry* of the world: rods and clocks.

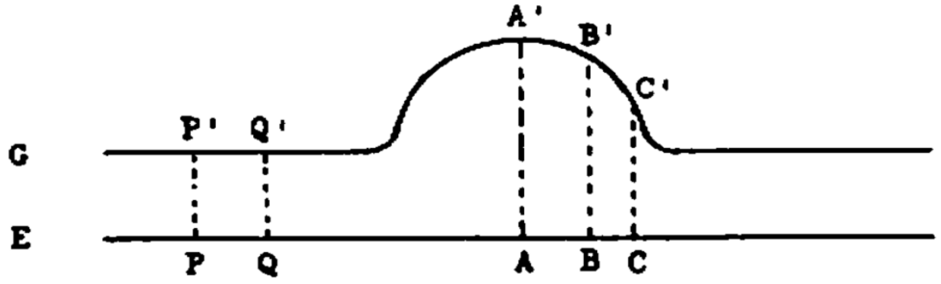
Geometric holism based on Poincaré’s equivalent proofs (1891): there are many empirically equivalent combinations of geometries [G] and “universal forces” (“universal effects”) [F]: {G+F , G’+F’ , G’’+F’’, ... }.

Conventionalism about space(time): ascertaining the physical geometry requires (in some way) a conventional choice.
→ Loosely associated with Poincaré, Duhem, Schlick, Carnap, and others, and Reichenbach (1926, Sec. 8).:

Theorem θ: “Given a geometry *G′* to which the measuring instruments conform, we can imagine a universal force *F* which affects the instruments in such a way that the actual geometry is an arbitrary geometry *G*, while the observed deviation from *G* is due to a universal deformation of the measuring instruments.”[1]

Theorem θ shows all geometries to be equivalent; it formulates the *principle of the relativity of geometry*. It follows that it is meaningless to speak about one geometry as the *true* geometry.

An effect (force) is universal iff it
1. cannot be screened off by insulating walls
2. acts equally on all materials/particle species
Otherwise it is a differential effect (or force)



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Weatherall & Manchak (2014): "The Geometry of Conventionality"

Can these trade-offs between geometry and universal forces *always* be made given the mathematics of modern spacetime theories?

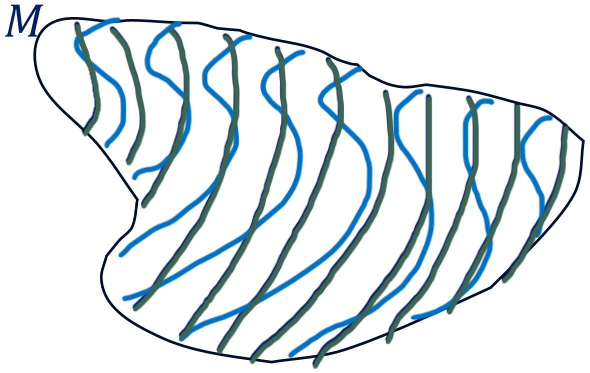
Weatherall & Manchak prove that – given a force field in the "standard sense" (i.e., acting on a test body in the geodesic equation like a rank-2 tensor) –

(Geometrised) Newtonian gravity: yes, this is always possible.
General relativity: no, this is not always possible.

Striking:
(i) Much-needed rigour to a conceptual debate;
(ii) Explicit theory-dependence;

However:
(iii) Are the assumptions justified? (Dürr & Ben-Menahem 2022, Mulder 2025a)
(iv) Little engagement with Reichenbach's project;
(v) No explanation / discussion of the discrepancy!

Spacetime (M, g, ∇)
with geodesics: $\xi^b \nabla_b \xi^a = 0$



Spacetime $(M, \tilde{g}, \tilde{\nabla})$
with geodesics: $\tilde{\xi}^b \tilde{\nabla}_b \tilde{\xi}^a = 0$

Spacetime $(M, \tilde{g}, \tilde{\nabla})$
with dynamics: $\tilde{\xi}^b \tilde{\nabla}_b \tilde{\xi}^a + (\mathbf{UF}) = 0$

Proposition 1: Newtonian gravity is underdetermined

<u>The non-relativistic case</u> (Prop. 1.)

Fix a classical spacetime (M, t_a, h^{ab}, ∇) and consider an arbitrary torsion-free derivative operator on M, $\tilde{\nabla}$, which we assume to be compatible with t_a and h^{ab}. Then there exists a unique antisymmetric field F^{ab} such that given any timelike curve γ with unit tangent vector field ξ^a, $\xi^n \nabla_n \xi^a = 0$ if and only if $\xi^n \tilde{\nabla}_n \xi^a = F^a_{\ n} \xi^n$, where $F^a_{\ n} = h^{am} F_{mn} \xi^n$.

In GNG (built on degenerate metrics t_a, h^{ab} and derivative ∇):

- There is no unique derivative operator compatible with t_a and h^{ab}.
  → A family of such operators exists.

- W&M show that any torsion-free connection $\tilde{\nabla}$ compatible with the same t_a and h^{ab} can be related to ∇ by a rank-2 anti-symmetric tensor field F_{ab}, interpreted as a force field (analogous to the Faraday tensor).

- This field generates accelerations just as in standard Newtonian gravity, so postulating it provides an acceptable universal force.

Proposition 2: GR is less susceptible to underdetermination

<u>**The relativistic case**</u> (Prop. 2.)

Let (M, g_{ab}) *be a relativistic spacetime, let* $\tilde{g}_{ab} = \Omega^2 g_{ab}$ *be a metric conformally equivalent to* g_{ab} *and let* ∇ *and* $\tilde{\nabla}$ *be the Levi-Civita derivative operators compatible with* g_{ab} *and* $\tilde{g}_{ab}$*, respectively. Suppose* Ω *is nonconstant. Then there is no tensor field* F_{ab} *such that an arbitrary curve* γ *is a geodesic relative to* ∇ *if and only if its acceleration relative to* $\tilde{\nabla}$ *is given by* $F^a{}_n\tilde{\xi}^n$*, where* $\tilde{\xi}^n$ *is the tangent field to* γ *with unit length relative to* $\tilde{g}_{ab}$. (W&M, p. 242-3)

Compute the difference tensor C (for $\nabla = (\tilde{\nabla}, C)$) for *conformally equivalent* spacetimes ($\tilde{g}_{bc} = \Omega^2 g_{bc}$):

$$C^a{}_{bc} = \frac{1}{2} g^{an}(\nabla_n g_{bc} - \nabla_b g_{nc} - \nabla_c g_{bn}) = \frac{1}{2\Omega^2}(g_{bc} g^{an}\nabla_n\Omega^2 - \delta_c{}^a\nabla_b\Omega^2 - \delta_b{}^a\nabla_c\Omega^2).$$

With normalization $g_{ab}\,\xi^a\xi^b = 1 = \tilde{g}_{ab}\tilde{\xi}^a\tilde{\xi}^b = g_{ab}\Omega^2\tilde{\xi}^a\tilde{\xi}^b \rightarrow \tilde{\xi}^a = \Omega^{-1}\xi^a$.

Geodesic equation $a^a = \tilde{\xi}^b\tilde{\nabla}_b\tilde{\xi}^a = \tilde{\xi}^b\nabla_b\tilde{\xi}^a + C^a_{bc}\tilde{\xi}^a\tilde{\xi}^b = \cdots = \cdots = \frac{1}{\Omega^3}(\xi^b\xi^c - g^{an})\nabla_n\Omega$.

If F_{ab} exists, then (on a ∇-geodesic): $\tilde{\xi}^b\tilde{\nabla}_b\tilde{\xi}^a = F^a_m\tilde{\xi}^m = \frac{1}{\Omega}\tilde{g}^{an}F_{nm}\xi^m = \frac{1}{\Omega^3}(\xi^b\xi^c - g^{an})\nabla_n\Omega$.

→ Together this leads to a contradiction → No F_{ab} exists for GR!

Comparative function: No “force-equivalent” to GR like NCT/NG

Dürr & Ben-Menahem (D&BM) say the restriction to a “standard force field” is **overly restrictive** and **conservative:**
→ Why not consider a broader notion of ‘interaction’, including rank-3 tensors $F^{a}{}_{bc}$?

Well, at least Fab has a *comparative* function.

Newton-Cartan theory has causally efficacious space and time …
… and is equivalent to Newtonian gravity, which is a “standard force” theory.

Is there a *Newtonian* force theory in flat space, equivalent to GR?

→ No such “standard-force-equivalent” exists for GR, analogous tothe existence of Newtonian gravity for Newton-Cartan theory.

	Geometric theory	**FORCE theory**
Non-relativistic	Newton-Cartan theory	Newtonian gravity
Relativistic	General relativity	

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Metric Degeneracy

are many empirically equivalent combinations of

In GR, the geometry uniquely determines the connection, so any change in connection requires a change in the metric, which cannot be captured by a standard force field.
In GNG, the connection is not uniquely determined by the metric structure, allowing one to reinterpret different connections (and thus different accelerations) as arising from universal forces.

But they don't dwell at length on why this difference arises.

Metric Degeneracy in non-relativistic gravity / Newton-Cartan theory

A metric is degenerate at a point $p \in M$ if the associated bilinear $g_p\colon T_pM \times T_pM \to \mathbb{R}$ is degenerate.

, at each point ppp, you can ask:
•Is gpg_pgp invertible as a linear map TpM→Tp∗MT_pM \to T_p^*MTpM→Tp∗M?
•Does it assign zero “length” to some non-zero vector in TpMT_pMTpM?
If yes, then ggg is degenerate at that point.

Sometimes we say that “the metric is degenerate” on a region or the whole manifold. This can mean:
•It is degenerate at every point (e.g. in Newton–Cartan theory).
•It is degenerate on a subset (e.g. on a horizon or singular surface).
•It is nowhere non-degenerate, so there is no globally invertible metric.

Thus: In

•Metric degeneracy is always determined pointwise: you check at each p∈Mp \in Mp∈M.
•Whether a global inverse metric exists depends on whether it’s non-degenerate at every point.

Why does there exist an F_{ab} for NCT but not for GR?

Why does there

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Unitism and Separatism

are many empirically equivalent combinations of

What spacetime structure bears on this debate?

Separatism allows for the underdetermination of Newton-Cartan

are many empirically equivalent combinations of

Degeneracy Supports Separatism (in some contexts):
For NCT, metric structure is degenerate and split:

- 1-form t_a defines absolute time and a rank-2 tensor h^{ab} defines spatial distances within time-slices.
- This naturally mirrors the separatist ontology: time and space are separate, primitive entities, not unified.
- There is no single unified metric uniting space and time into full 4D distances or intervals.

Unitism Requires a Non-Degenerate Metric
The main motivation for unitism, especially in the context of special and general relativity, is the invariance of the spacetime interval, defined by a non-degenerate metric:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

This metric fuses space and time into a single geometric structure—something that breaks down if $g_{\mu\nu}$ is degenerate.
→ Degeneracy implies you cannot define invariant intervals or may need to define causal structure in some other way.

Conventionalism and incompleteness: Kaluza-Klein & Unification

Can this be generalised?

Unitism, Unity, Unification…

Can this be generalised?

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Conclusion

Can this be generalised?

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