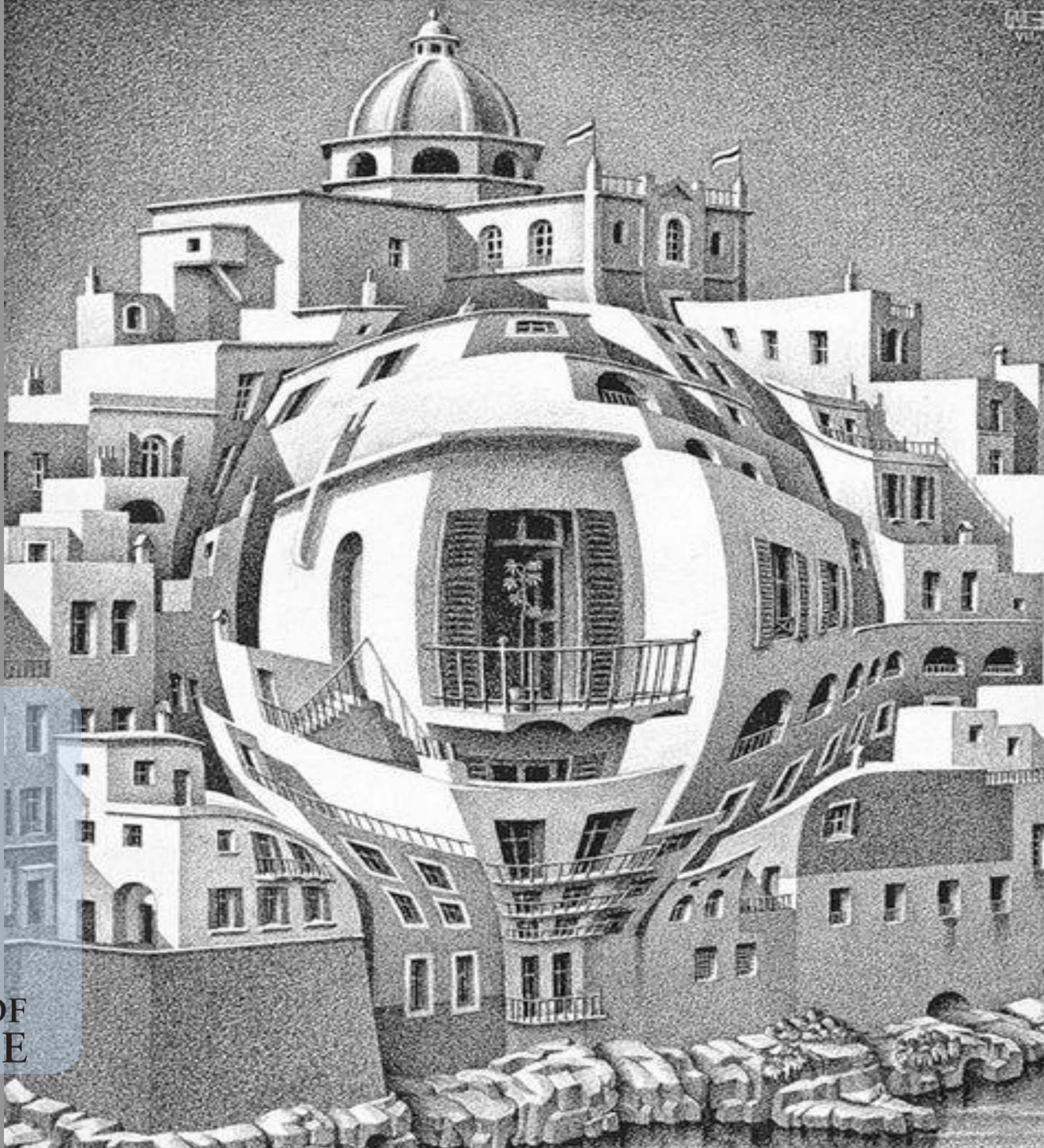
Is spacetime curved?
Underdetermination of general and teleparallel relativity
(joint work with James Read)

Ruward Mulder
BSPS 2023
Bristol, 6 July

UNIVERSITY OF CAMBRIDGE



Underdetermination:
Teleparallel gravity and torsion

Knox: reverse-engineering and
inertial structure

A problem of visualisation?
Intrinsic and extrinsic torsion

A proposal: Lie flow as
“Direction curvature”

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Teleparallel gravity I/II

Where geodesics come apart

There exists an empirically equivalent theory to GR, called the Teleparallel Equivalent of General Relativity (TEGR)

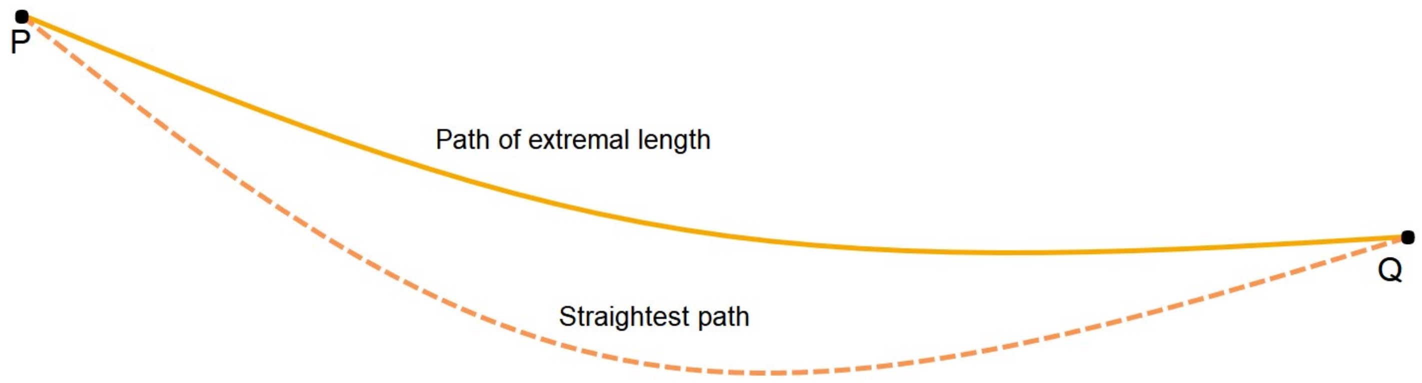
TEGR employs an **antisymmetric connection** $\tilde{\nabla}$ instead of a symmetric one $\bar{\nabla}$.

In GR: Levi-Civita connection $\bar{\nabla}$ is uniquely metric-compatible.
In TEGR: Weitzenböck connection $\tilde{\nabla}$ is metric-compatible but not unique.

They are related via the contorsion tensor: $K^{\rho}{}_{\mu\nu} = \bar{\Gamma}^{\rho}_{\mu\nu} - \tilde{\Gamma}^{\rho}_{\mu\nu}$.

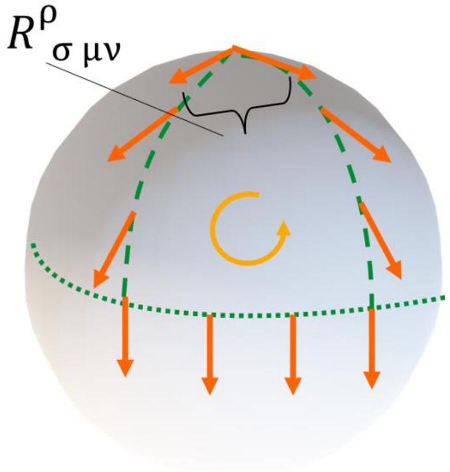
Two notions of geodesic come apart:

- **Metric-geodesics**: paths of extremal length, through $g_{\mu\nu}$.
- **Affine-geodesics** (or auto-parallels): straightest paths, through ∇.



Teleparallel gravity II/II

Should we commit to curvature as a real property of spacetime?



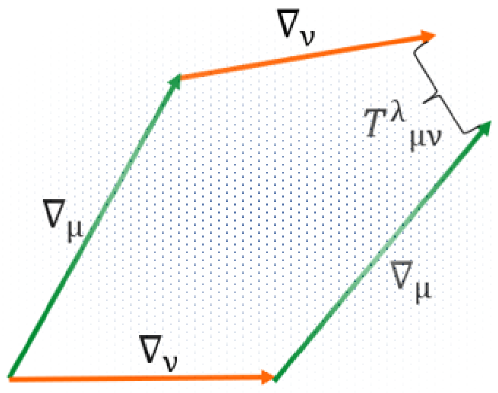
Consequence of having an antisymmetric affine connection:

- Riemann tensor $R^{\rho}{}_{\sigma\mu\nu} \equiv 0$, thus **no (textbook) curvature**
- Torsion tensor $T^{\rho}{}_{\mu\nu} \neq 0$ is non-zero , thus the spacetime is **torsioned**.

This leads to non-closure of parallelograms.

The empirical equivalence is at the *dynamical* level:

$$S_{TEGR} = \int dx^{\mu} \sqrt{-g}\, T = -\int dx^{\mu} \sqrt{-g}\, R - 2\nabla^{\nu} T^{\rho}{}_{\nu\rho} = S_{EH} + \text{boundary term.}$$



The stakes: is spacetime really curved?

Sebastian Bahamonde *et al* (2022). ``Teleparallel gravity: from theory to cosmology."
Ruben Aldrovandi & José G. Pereira (2013). *Teleparallel Gravity: An Introduction.*

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Symmetry and reverse-engineering GR (I/II)

The tetrad and metric formulations

Important paper: Knox (2011) ``Newton-Cartan theory and teleparallel gravity: the force of a formulation.“ → TEGR is merely a reformulation of GR.

Multiple overlapping arguments: we distill three “Problems”.

Teleparallel gravity is usually set in the **tetrad formalism** : $e^a_\nu V^\nu = V^a$.
Main teleparallel quantities are commonly expressed with them: $\tilde{\Gamma}^\rho_{\mu\nu} = e^\rho_a \partial_\nu e^a_\mu$.

Prominently the metric: $g_{\mu\nu} = \eta_{ab} e^a_\mu e^\rho_\nu$.

But: the tetrad formalism can (freely) be introduced in GR;
and although tetrads make torsion easier to recognise, this is not necessary!

	Metric formalism	Tetrad formalism
Symmetric ∇	$dx^\mu \sqrt{-g}\,\mathrm{R}$	$dx^\mu \sqrt{-e}\,R$
Antisymmetric ∇	$dx^\mu \sqrt{-g}\,T$	$dx^\mu \sqrt{-e}\,T$

#1 Problem of the Shy Metric Tensor

“In teleparallel gravity [the metric] does not appear in the formalism of the theory. Nonetheless, it is worth noticing that it has been hiding in the shadows all along, closely tied to the tetrad field. In fact, $g_{\mu\nu}$ is still used to raise and lower indices, just as it is in GR. One might therefore have doubts that teleparallel gravity really postulates a different ontology; the old entities from GR appear to be waiting in the wings.” (Knox 2011, pp. 273--274)

Jose Beltrán Jiménez, Lavinia Heisenberg and Tomi S. Koivisto (2018). ``Teleparallel Palatini theories.”
Salvatore Capozziello, Vittorio De Falco & Carmen Ferrara (2022). ``Comparing Equivalent Gravities: common features and differences.”

Symmetry and reverse-engineering GR (II/II)

The teleparallel derivative and conserved quantities

Another reverse-engineering problem:
Conserved quantities appear under the **teleparallel derivative**:

$$D_\mu j_a^\mu \equiv \partial j_a^\mu + (\tilde{\Gamma}^\rho_{\mu\nu} - K^\rho{}_{\mu\nu}) j_a^\nu.$$

This closely mimicks the Levi-Civita connection of GR.

This Problem has a real bite: the Weitzenböck connection does not play as central a role in TEGR as does the Levi-Civita connection in GR.

But TEGR can stand on its own two legs! GR is need not be mentioned.

Besides: reverse-engineering not surprising, as there is supposed to be symmetry.

Why more **natural**? → familiarity with a **thick notion of inertial structure.**

#2 Problem of Conserved Quantities

It would be more natural *if conserved quantities would come out without the use of contortion. Hence, one gets the impression that it is the Levi-Civita connection that is doing all the work, not the Weitzenböck connection, establishing precedence of GR.* (rephrasing of (Knox 2011, p. 272).)

The absence of inertial structure

The non-vanishing of the Weitzenböck connection coefficients and inertial functionalism

P1: A piece of geometrical structure can be regarded as representing spacetime only if it allows one to define (a structure of) local inertial frames

P2: A necessary condition for a derivative operator to define a structure of local inertial frames is that it has connection coefficients which can be made to vanish (at least locally)

P3: The components of the Weitzenböck connection cannot be made to vanish (even locally)

C: The Weitzenböck connection cannot be regarded as representing spacetime.

We believe there are other options available, corresponding to denying **P1-P3**.

#3 Problem of Non-vanishing Connection Coefficients

In GR, the inertial frame is where the coefficients of the Levi-Civita connection are zero. For the Weitzenböck connection this does not happen for any reference frame. (rephrasing of (Knox 2011, p. 273).)

David John Baker (2019). ``On Spacetime Functionalism.” Knox’s inertial spacetime functionalism

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Deny **P1**: Inertial frames need not be so central to the functional role of spacetime
There are criteria to construct a functional role for spacetime that have nothing to do with inertial structure (cf. David Baker 2019).

David John Baker (2019). ``Knox's inertial spacetime functionalism."

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Deny **P2**: We need not adopt such a **thick notion** of inertial structure, where *force-free bodies* necessarily follow the trajectories given by the "kinematically fundamental" objects of the theory?

In TEGR, bodies follow paths given by *combinations* of kinematically fundamental objects: Weitzenböck *and* contorsion
→ The Weitzenböck connection satisfies a **thin notion** of inertial structure, where straight paths are defined by the "kinemetically fundamental" affine connection.

It seems the thick notion is not theory-neutral and stems from familiarity with GR.

<u>Read</u>, James & Tushar Menon (2021). "The limitations of inertial frame spacetime functionalism."

The absence of inertial structure

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C: The Weitzenböck connection cannot be regarded as representing spacetime.

Deny **P3**: One could adopt anholonomic coordinates
→ Yet, (Knox 2013) argues that anholonomic coordinates are not physical (non-linearity, path-dependence).

Eleanor Knox (2013). ``Effective spacetime geometry"

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Visualising torsion

Is teleparallel gravity somehow metaphysically problematic?

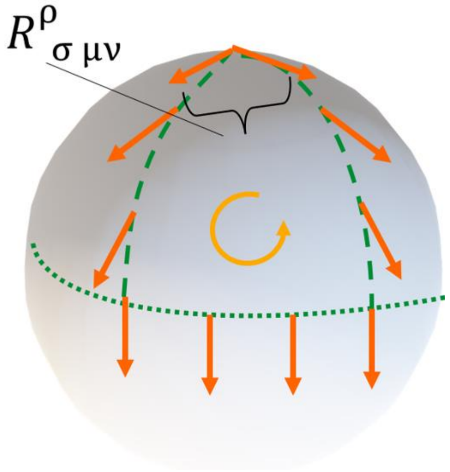
A common complaint is that torsion is too hard to visualise!

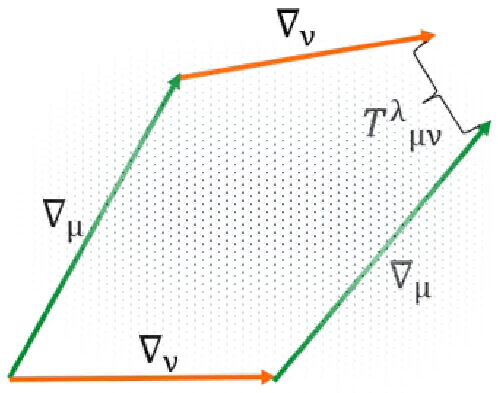
And even that *therefore* it is not a good theory to adopt:

The Problem of Visualisability

Curvature as a property of spacetime can be visualised, whereas torsion cannot. Assuming that visualisability is important for theory choice, GR and TEGR are not on a par.

Distinction between intrinsic and extrinsic torsion is essential.





For visualisation as an epistemic virtue, see:
Magdalena Kersting (2021). "Visualizing Four Dimensions in Special and General Relativity."
Henk W. De Regt (1997). "Erwin Schrödinger, Anschaulichkeit, and quantum theory."

Extrinsic and intrinsic torsion

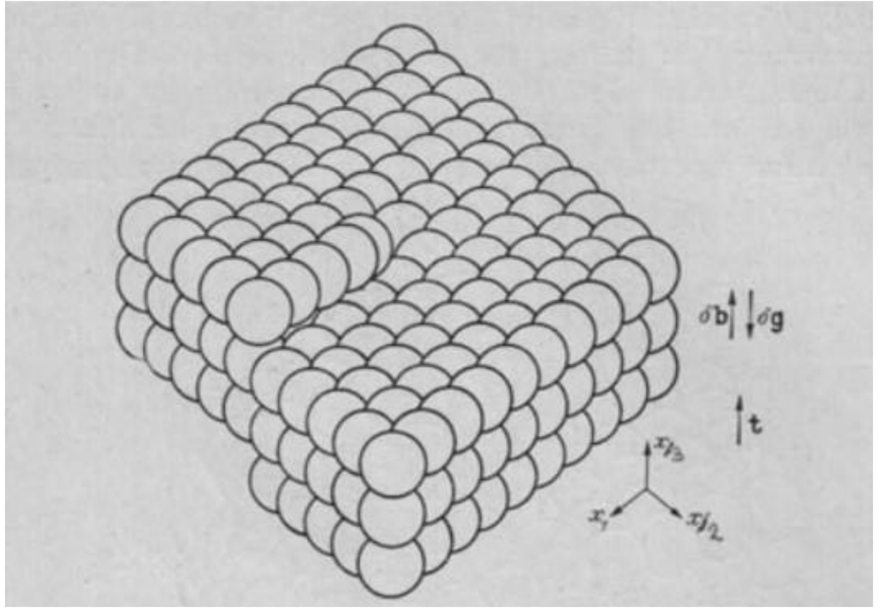
Just as easy or hard to visualisable as curvature

- <u>Extrinsic visualisations:</u>

Easy for curvature: embedding in 3d Euclidean space.

This has been done for torsion too: various work on Hehl, Obukhov, Lazar.

For example on crystal structures with torsion: dislocations.



- <u>Intrinsic visualisation:</u>

This is perhaps impossible, already for curvature.

But Reichenbach (p.~55): we have insufficient *experience* of non-Euclidean geometries.

The untrained automobile driver sees the images in the rearview mirror as distorted, changing shape. Not so for the trained driver!

→ Conclusion: for visualisability, curvature and torsion are on a par.

Friedrich Hehl & Yuri Obukhov (2007). "Élie Cartan's torsion in geometry and in field theory, an essay."
Markus Lazar (2002). ``An elastoplastic theory of dislocations as a physical field theory with torsion."
Hans Reichenbach (1928). *The Philosophy of Space and Time*.

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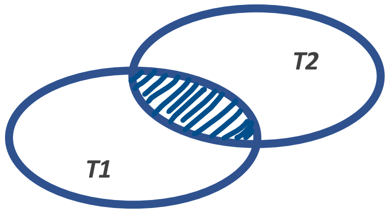
Direction curvature

Is there a common core solution to the underdetermination?



Can the common core approach help?

I think yes: commit to what I call "**direction curvature**", i.e., **Lie flow**.

Using a completely general (symmetric+antisymmetric) connection:

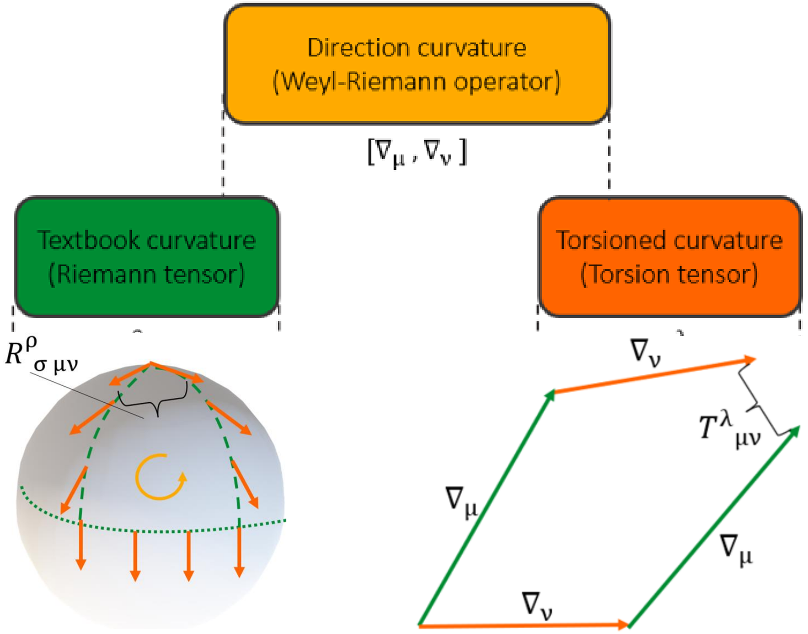
$$[\nabla_\mu, \nabla_\nu]\, V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\sigma{}_{\mu\nu}\, \nabla^\sigma V^\rho .$$

Make a conventional choice how to set up the straight line to get physics of the ground.

Giving up on the straight line is certainly not perspicuous!

Relates to sophistication/perspicuity debate: **loss of affine structure**.

→ logically weaker statements about nature, but no more underdetermination.

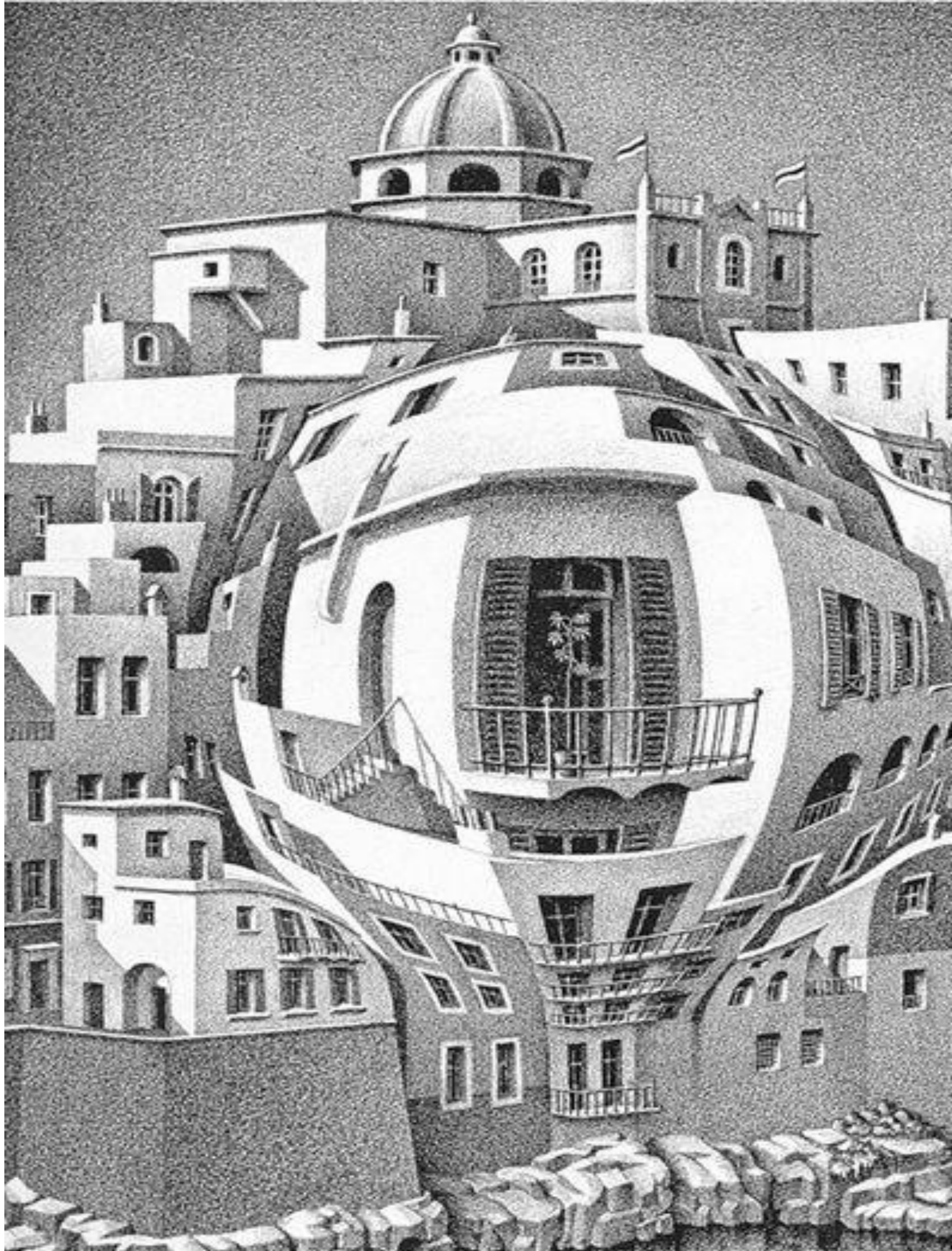


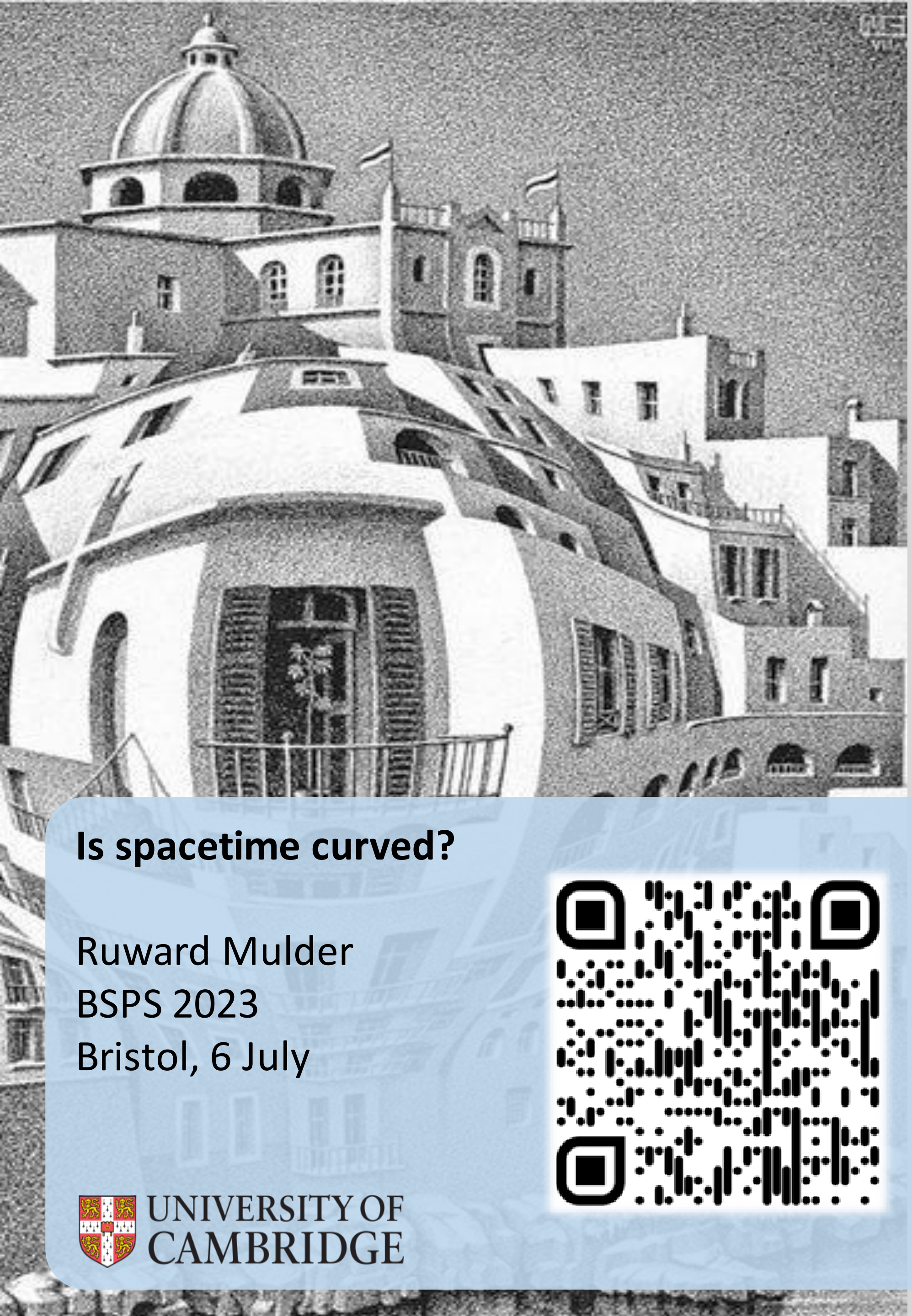
Hermann Weyl (1918). "Gravitation und Elektrizität."
Thomas Møller-Nielsen (2017). ``Invariance, interpretation, and motivation."
Neil Dewar (2019). ``Sophistication about symmetries."

Conclusion

- There exists an empirically equivalent alternative to GR with no curvature.
- This prompts us to reconsider ontological commitment to curvature.
- Some reasons to prefer GR to TEGR on independent grounds see insufficient: there are no convincing Problems of reverse-engineering, or missing inertial structure.
- Perhaps there are other reasons: e.g., simplicity…
- Nor is the Problem of Visualisation convincing:
  → extrinsic torsion is just as visualisable as textbook curvature
  → intrinsic torsion is just as non-visualisable as textbook curvature.
- I propose: commit to common core, namely direction curvature, as the non-commutation of covariant derivates / the Weyl-Riemann operator:

$$[\nabla_\mu, \nabla_\nu] \neq 0.$$





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