With only a few colours

The AB effect and the potentials

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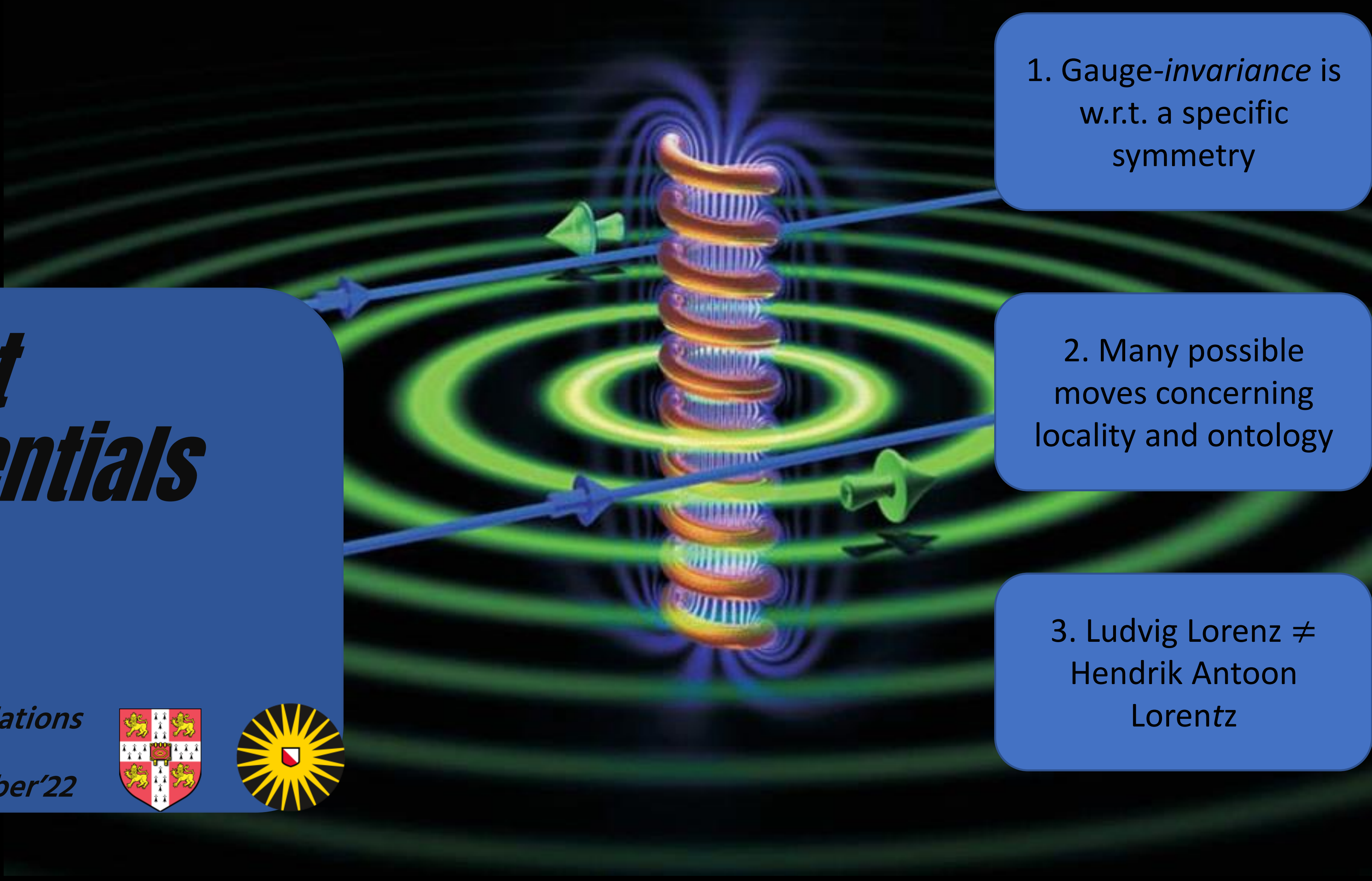


1. Gauge-*invariance* is w.r.t. a specific symmetry

2. Many possible moves concerning locality and ontology

3. Ludvig Lorenz ≠ Hendrik Antoon Lorentz

How real are the electromagnetic potentials?

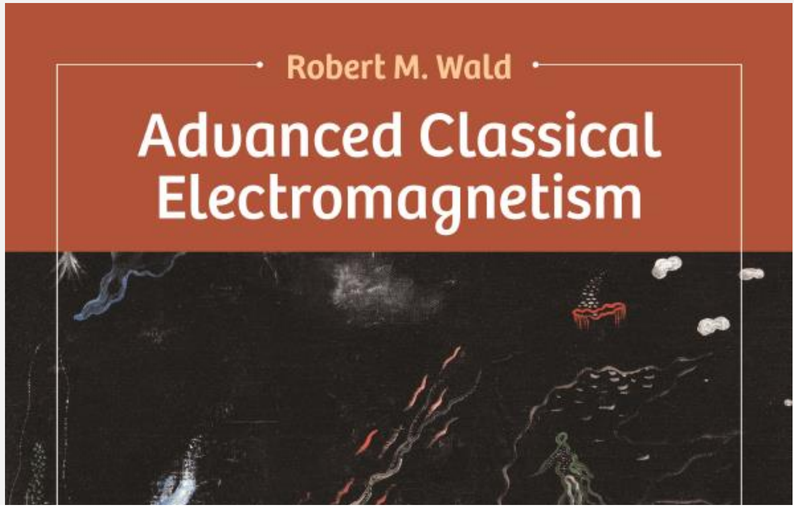
Underdetermination of (E,B)-theory and (ϕ,A)-theory

Situation. Textbook interpretation of the EM potentials is the *Fields View*: ϕ and A are descriptively redundant – they are only `real' insofar as they give rise E and B. The Aharonov-Bohm effect challenges this: the potentials seem physically effective beyond E and B.
→ The *Potentials View* has gained ground, *viz*. Bob Wald (2022).

Complication. If ϕ and A are real, it is not clear how to understand this: there is gauge-underdetermination. Also, there are tensions between interpretative moves concerning gauge-equivalence and different shades of locality. This is not well treated in lecture halls.

Question. If the potential fields are `real', in what sense are they real?

Claim. They are still partly descriptively redundant. The physical part can be fixed by narrowing down (`gauge fixing') the gauge-equivalence class of ϕ and A. This is justified through a metaphysical criterion. I argue for the criterion of signal locality, leading to the Lorenz gauge.



Complication: If the potential fields are `real', it is not clear in what sense!

Claim: They are still partly descriptively redundant – the real part is fixed by the Lorenz gauge

The AB effect
Non-local interactions?

Gauge-equivalence
with respect to which symmetry?

Bell locality to the rescue?
No!

The Lorenz gauge
'If locality is your criterion, stick to it'

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The Aharonov-Bohm effect
Challenging traditional (Field) views

The Aharonov-Bohm experiment is the standard two-slit experiment with an additional solenoid.

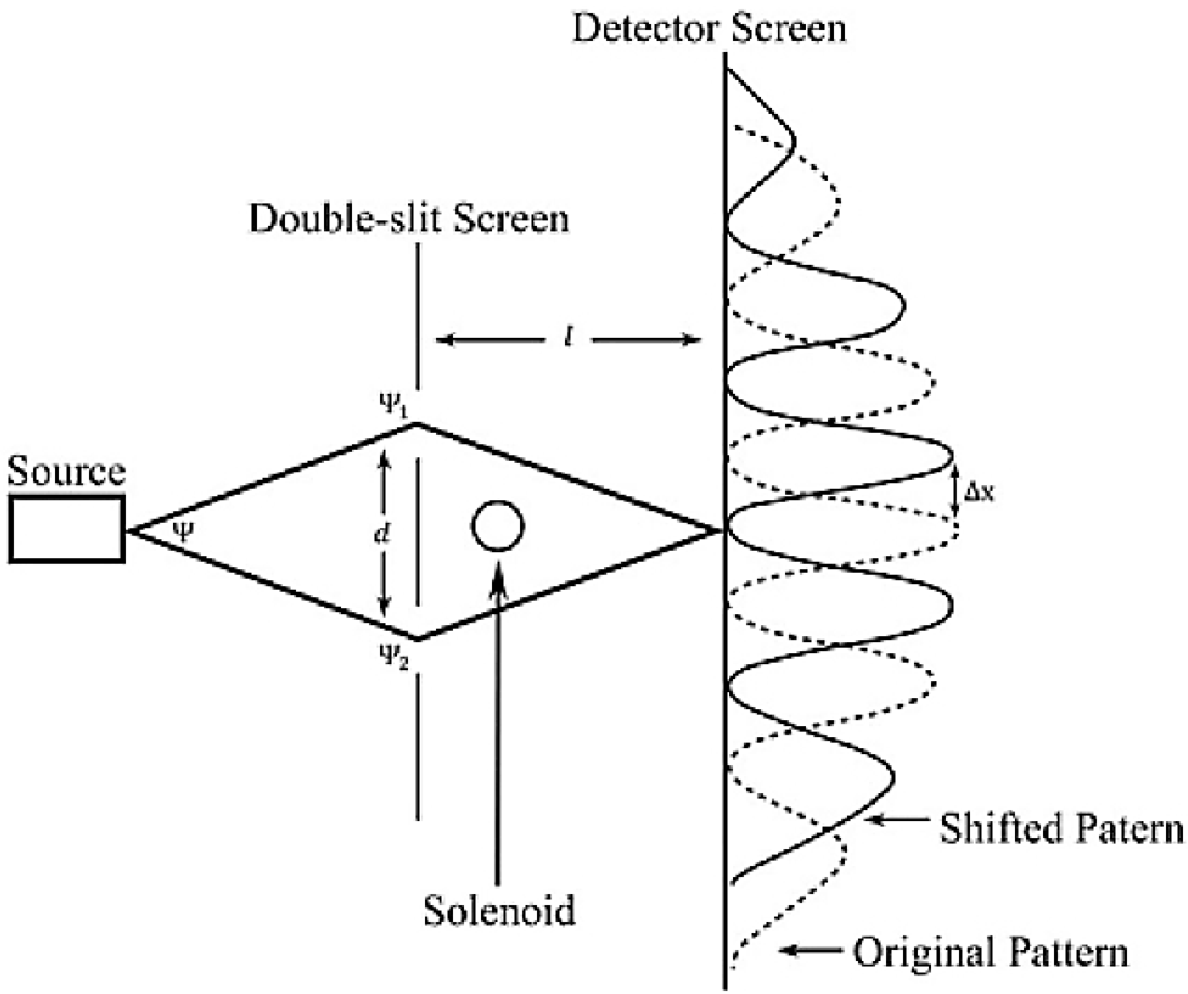
Switch the current in the solenoid on and the interference pattern changes! The double-slit pattern is shited upwards (or downwards):

$$\Delta x = -\frac{\lambda_B l}{2\pi d}\frac{q}{\hbar}\oint_C \mathbf{A}\cdot d\mathbf{x}$$

How to explain this effect !?

Well-known result from EM: the magnetic B-field of an (infinite) solenoid is confined to the interior of the solenoid: it vanishes outside.

The vector potential **A** does not vanish outside!



The Aharonov-Bohm effect
Physical efficacy and indispensibility

Aharonov & Bohm (p.490) argue:

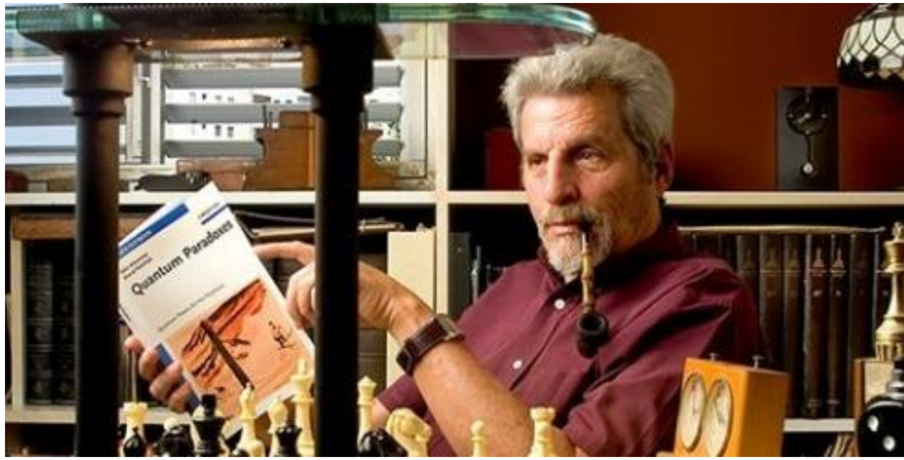
> *Of course, our discussion does not bring into question the gauge invariance of the theory. But it does show that in a theory involving only local interactions (e.g., Schrodinger's or Dirac's equation, and current quantum-mechanical field theories), the potentials must, in certain cases, be considered as physically effective, even when there are no fields acting on the charged particles.*

Also! The potentials seem to be unavoidable in quantum mechanics: the Schrödinger equation necessarily deals with a potential and not forces!

$$H = \frac{1}{2m}(\mathbf{p} - q\mathbf{A})^2$$

So perhaps the vector potential **A** is real!

But, wasn't this potential field subject to a gauge transformation?



Aharonov, Y., Bohm, D.: Significance of electromagnetic potentials in the quantum theory. Phys. Rev. **115**(3), 485 (1959)

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Gauge-equivalence classes

Gauge-underdetermination as a result from reagrding the potentials as real

The potential fields will give the same results for E and B, regardless of the value of the gauge parameter in the *standard* (but tailor-made) gauge equivalence class.

Trade-off: the conversion of gauge degrees of freedom into physically real degrees of freedom!

This is not so strange: consider the Wide Equivalence Class – already the AB effect is incorporated.

$$\phi \mapsto \phi' := \phi - \frac{\partial \chi}{\partial t}$$
$$\mathbf{A} \mapsto \mathbf{A}' := \mathbf{A} + \nabla \chi$$

$$\phi \mapsto \phi' := \phi - C_0$$
$$\mathbf{A} \mapsto \mathbf{A}' := \mathbf{A} + \mathbf{C}$$
$$\text{subject to } \nabla \times \mathbf{C} = 0$$
$$\text{and } \nabla C_0 = \partial \mathbf{C}/\partial t$$

Crudest way to break gauge symmetry: stiplate that there is ONE TRUE GAUGE (Maudlin 1998).

Maybe that’s a bit too much: how to justify such ontological extravagance?

Perhaps locality can help to find a middle way?

Maudlin, T.: Healey on the Aharonov-Bohm effect. Philos. Sci. **65**, 361 (1998)

A Spectrum of Narrowing Equivalence classes

(ϕ,A)-theories on a spectrum, each with their own locality interpretation

The potentials view

Singleton Set
(even the unit is fixed)

One True Gauge
(no gauge freedom)

E.g. Coulomb gauge
or Lorenz gauge
+ Temporal gauge

subject to $\nabla^2 \chi = 0$

Lorenz Equivalence Class
(signal locality)

$$\phi \mapsto \phi' := \phi - \frac{\partial \chi}{\partial t}$$

$$\mathbf{A} \mapsto \mathbf{A}' := \mathbf{A} + \nabla \chi$$

subject to $\nabla^2 \chi = \frac{1}{c^2} \frac{\partial^2 \chi}{\partial t^2}$

Narrow Equivalence Class
(local interactions)

$$\phi \mapsto \phi' := \phi - \frac{\partial \chi}{\partial t}$$

$$\mathbf{A} \mapsto \mathbf{A}' := \mathbf{A} + \nabla \chi$$

for any χ

Wide Equivalence Class
The fields view

$$\phi \mapsto \phi' := \phi - C_0$$

$$\mathbf{A} \mapsto \mathbf{A}' := \mathbf{A} + \mathbf{C}$$

subject to $\nabla \times \mathbf{C} = 0$
and $\nabla C_0 = \partial \mathbf{C} / \partial t$

There is more space to explore here! Perhaps by YOU?!

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Bell Locality

Can Bell save us from gauge-underdetermination?

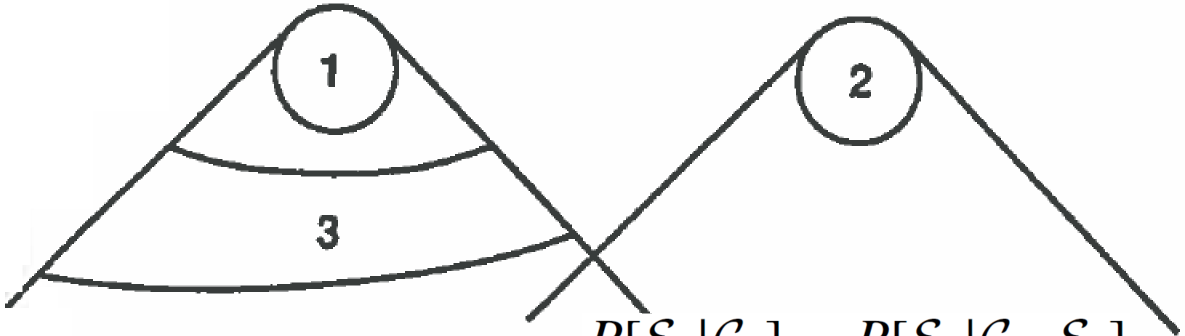
A theory is Bell local iff events in 2 are *irrelevant* for predictions about events in 1, given that one knows everthing in region 3.

→ Not really helpful to the AB set-up!

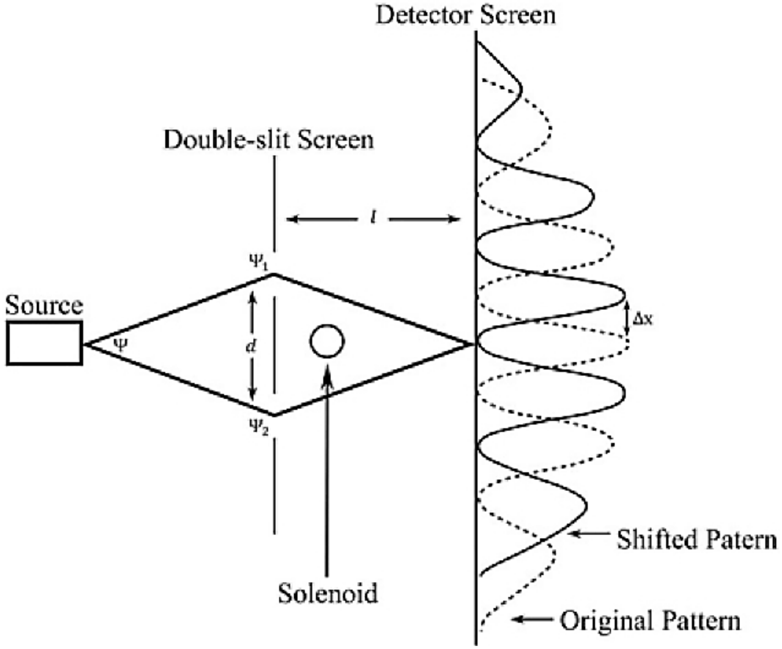
The electron beam comes *arbitrarily close* to the solenoid, so that two events 1 and 2 involving the electron cannot be spacelike separated.

The electrons need to form a closed loop and electrons are slower than light.

The Aharonov–Bohm set-up involves a single system.



$$P[\mathcal{E}_1|\mathcal{C}_3] = P[\mathcal{E}_1|\mathcal{C}_3, \mathcal{E}_2]$$



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Consider the Lorenz gauge $\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$

Signal locality as a way to cut down on gauge-underdetermination

Locality is the original motivation for the various metaphysical moves when interpreting the AB effect.

I propose that signal locality is a criterion that leads to a perspicuous ontology. This is close to intuitions about 'locality':

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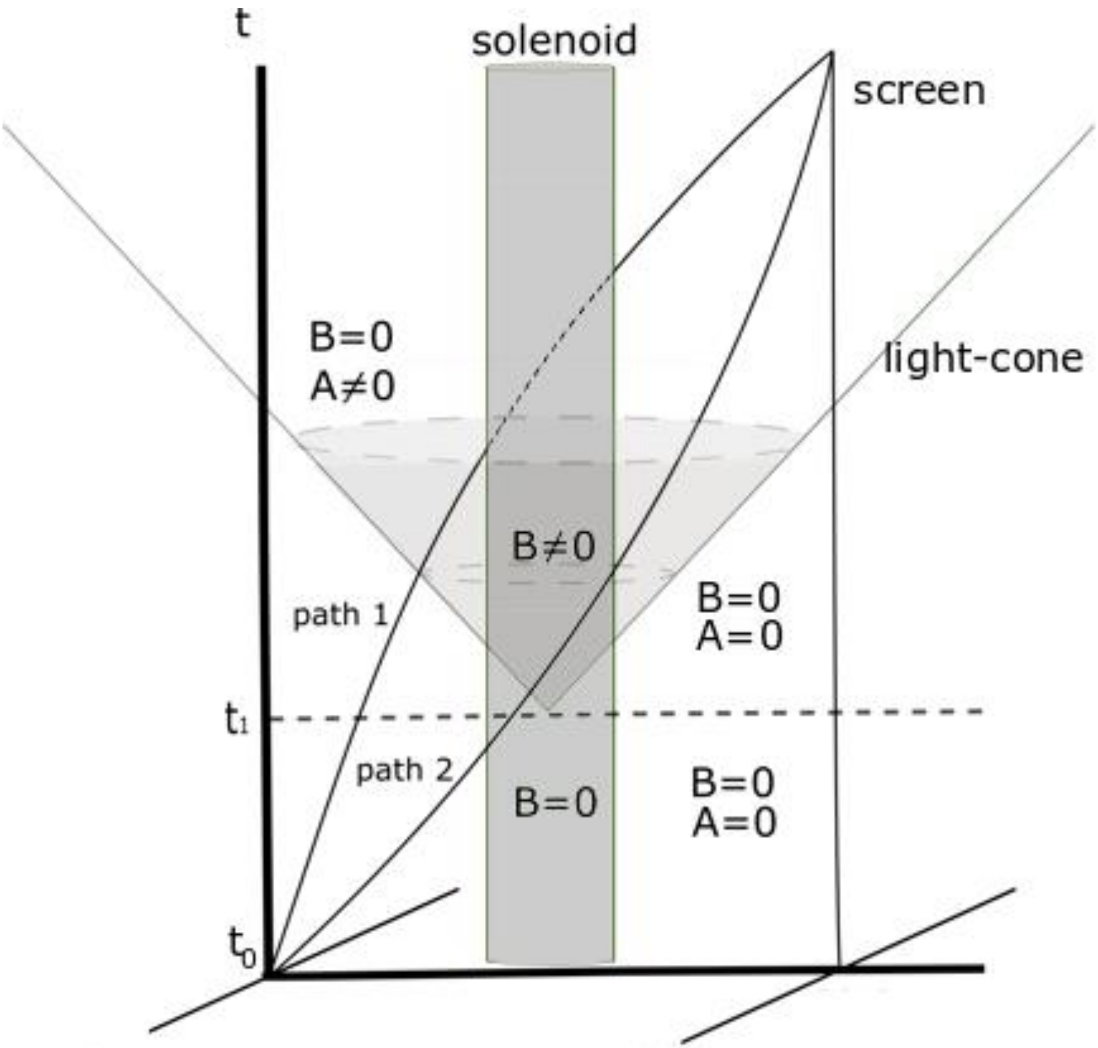
> causal influences that bodies exert on one another propagate at some finite speed propagate via a physical process

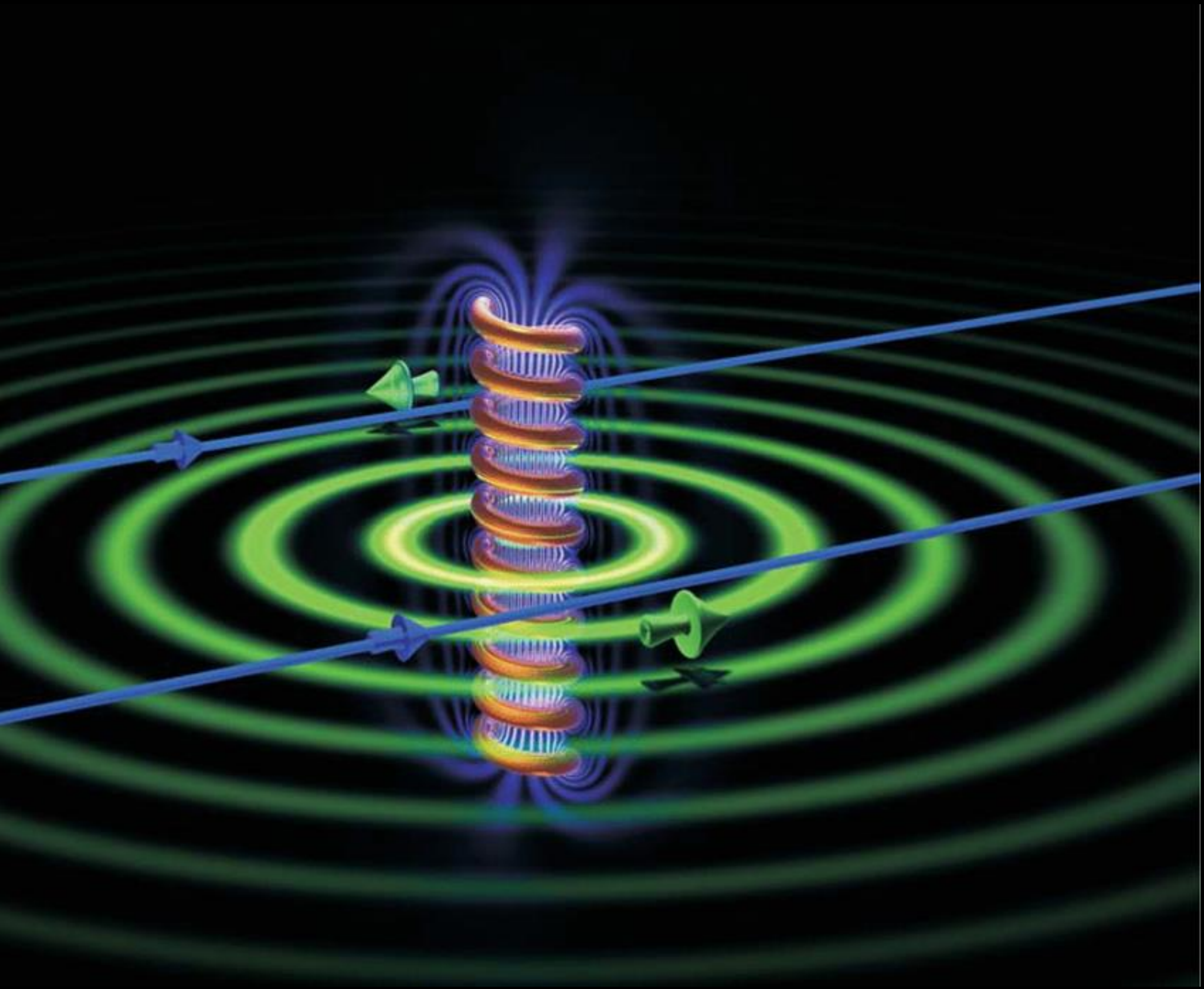
Constraining the gauge parameter to $\nabla^2 \chi = -\frac{1}{c^2}\frac{\partial^2 \chi}{\partial t^2}$

Leads to wave equations for the potentials: $\nabla^2 \phi = \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2}$ & $\nabla^2 \mathbf{A} = \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2}$

→ a continuous (in time) physical process through space.

Thus signal locality is satisfied! Very close to the locality of E and B.





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Some definitions

Point-like interaction: Two things that interact can be defined within arbitrarily small spacetime regions. They can couple in those regions.

Signal locality: All causes of an event propagate via a physical process that is continuous through space.

Separability: Divide some system into spatiotemporally divided subsystems. You can reconstruct the original system from the properties of the subsystems alone.