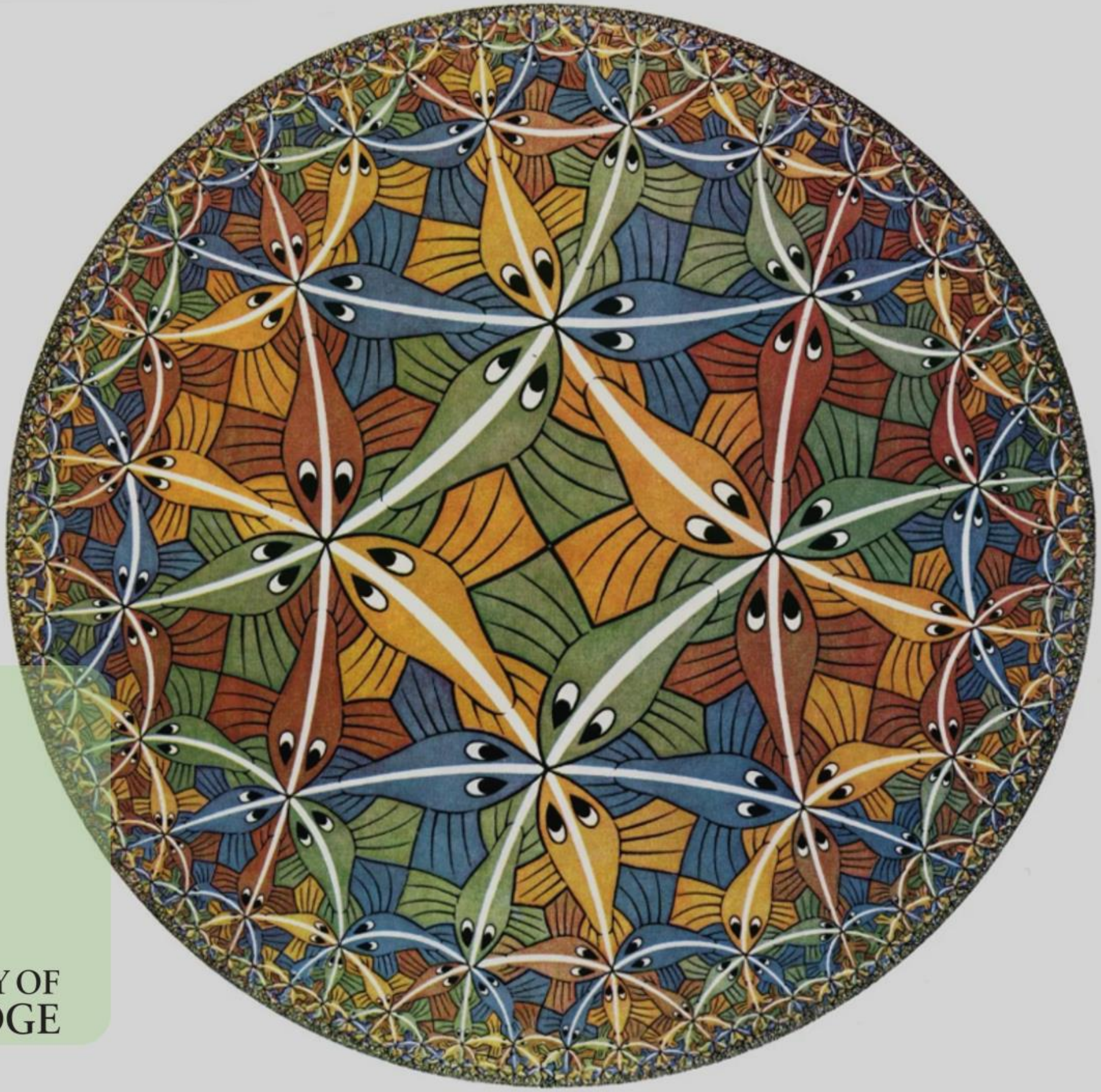


Is general relativity safe for conventionalism? Loopholes to Weatherall & Manchak's proof

Ruward Mulder

Foundations 2023
Bristol, 8 July

UNIVERSITY OF CAMBRIDGE

The idea behind (one kind of) geometric conventionalism

Trading-off geometry and forces

What is it for spacetime theories / models to have the same empirical content?
→ Same trajectories of test particles.

Trade-off equation (Reichenbach's Theorem Θ):

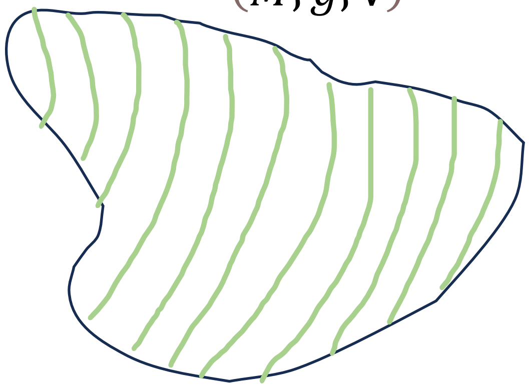
$$\xi^b \nabla_b \xi^a = \xi^b \tilde{\nabla}_b \xi^a + G_b{}^a \, \xi^b,$$

for universal "force" $G_{ab} \coloneqq g_{ab} - \tilde{g}_{ab}$.

This is a funny force, or rather *effect,*
1. for which there are no insulating walls
2. which acts on all materials/particle species equally.

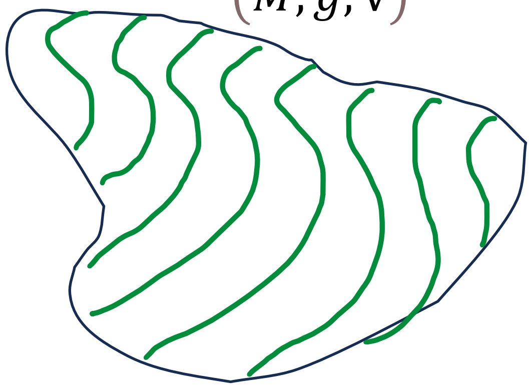
Given that we agree this constitues different models / theories, is it true?

(M, g, ∇)



Dynamics: $\xi^b \nabla_b \xi^a = 0$

$(M, \tilde{g}, \tilde{\nabla})$



Dynamics: $\xi^b \tilde{\nabla}_b \xi^a + \text{universal force} = 0$

Reichenbach (1928). *Raum und Zeit.*

W&M: rigourous trading of geometry & force

A proof makes assumptions

Looking at one assumption: Conformal equivalence

Conclusion: the burden of proof…

W&M: rigourous trading of geometry & force

A proof makes assumptions

Looking at one assumption: Conformal equivalence

Conclusion: the burden of proof...

Weatherall & Manchak: ``the geometry of conventionality”

Can one accommodate different geometries by postulating a new force field?

Newtonian gravity: yes, the trade-off *is* possible.

General relativity: no, the trade-off is *not* possible.

Counterexample for $\Omega_n(t, x, y, z) = x^2 + \frac{1}{n}$,
with the curve passing through $\left(0, \frac{1}{\sqrt{n}}, 0, 0\right)$.

Good result that advances the debate:
(i) Theory-dependence
(ii) Rigourous input to conceptual debate.

But also:
(iii) Presentation gives (in my opinion) a false air of exhaustiveness
(iv) Neglected opportunity for more conceptual discussion and context.

The relativistic … **n 2.)**

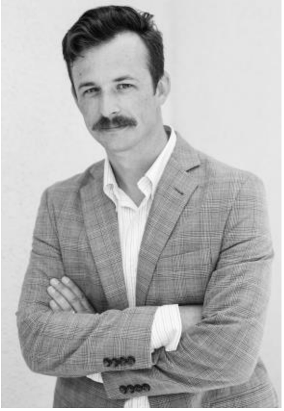
Let (M, g_{ab}) be a relativistic sp… $= \Omega$ … c conformally equivalent to g_{ab}… $\tilde{\nabla}$ b… a derivative operators compati… $\tilde{g}$ … Suppose Ω is nonconstant. Ther… or fi… t an arbitrary curve γ is a geodesic relative to ∇ if and only if its acceleration relative to $\tilde{\nabla}$ is given by $G^a{}_n\xi^n$, where ξ^n is the tangent field to γ with unit length relative to $\tilde{g}_{ab}$. (W&M, pp. 242--243)

Jim Weatherall & JB Manchak (2014). “The Geometry of Conventionality.”

Results by Weatherall & Manchak

Newtonian gravity is safe for conventionalism – relativity is not?



Patrick Dürr & Yemima Ben-Menahem challenge the proof by:

- Listing assumptions; giving counterexamples; and proposing selective realism about geometry (conventionalism).

My claims:

- Differential geometry is very malleable: lots of trade-offs to exploit
- A peaceful programmatic approach would be more fruitful: systematically challenging the W&M assumptions and prove further theorems
- Assumption of physical force, and the burden of proof…
- Assumption of conformal symmetry: there is some talking across purposes (and it's Reichenbach's fault?).

Patrick Dürr & Yemima Ben-Menahem (2022). "Why Reichenbach wasn't entirely wrong, and Poincaré was almost right, about geometric conventionalism."

Jim Weatherall & JB Manchak (2014). "The Geometry of Conventionality."

No one is arguing about conventionalism

This is about underdetermination (UDD) of empirically equivalent models

Conventionalism is:

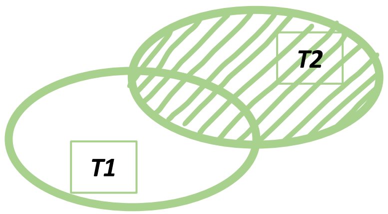
- A **response** to underdetermination
- A common core approach: **selective realism** about the formalism, denying truth value of parts of the formalism.

There is no dispute over truth values or the nature of the convention.

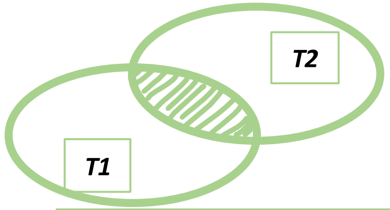
The debate is *not* over conventionalism
→ but over whether GR is *safe for conventionalism*.

Given a theory, are these trade-offs between force and geometry possible?

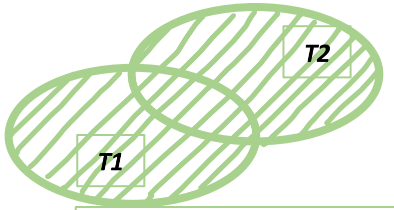
It is important whether this is UDD of models or UDD of theories.
→ Both are possible!



(a) Prefer on independent grounds



(b) Commit to the common core



(c) Explore pluralistic options

W&M: rigourous trading of geometry & force

A proof makes assumptions

Looking at one assumption: Conformal equivalence

Conclusion: the burden of proof…

The ingredients

Every proof has assumptions, but they may be too strong…

UDD of models?

Dürr and Ben-Menahem give:

- FORCE. a compensating universal force should be a 2-tensor $G_b{}^a$,
- CONF. the alternative metric is conformally related to the standard metric: $\tilde{g}_{ab} = \Omega^2 g_{ab}$
- NORM. $\tilde{\xi}^b$ is a vector of unit norm with respect to the **new** geometry: $\tilde{g}_{ab}\tilde{\xi}^a\tilde{\xi}^b = 1$
- ALT-ACC. A geometric alternative must accommodate a massive particle's acceleration

UDD of theories?

- RIEM. Geometric alternatives must employ Riemannian geometry

  I would break this one up:
  - SYMM. The affine connection is symmetric
    (~SYMM should include teleparallel equivalent to GR)
  - COMP. The affine connection is metric compatible
    (~COMP should include symmetric teleparallel equivalent to GR)

  I would add:
- DIM. We work in 3+1 dimensions. (~DIM could accommodate, e.g., Kaluza-Klein theory.)

Sebastian Bahamonde et al.(2021).
Teleparallel Gravity: From Theory to Cosmology

W&M: rigourous trading of geometry & force

A proof makes assumptions

Looking at one assumption: Conformal equivalence

Conclusion: the burden of proof…

CONF

Why would we confine ourselves to conformally equivalent spacetimes?

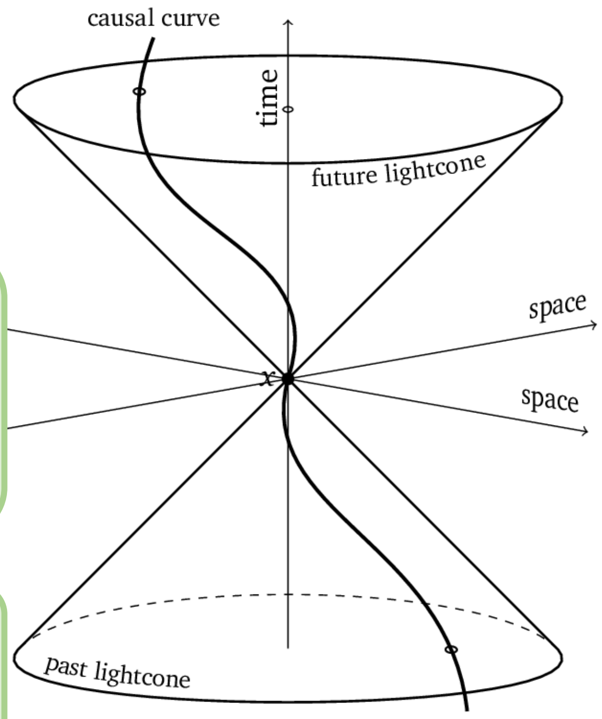
W&M restrict the proof to conformally equivalent spacetimes.

At first sight, this seems strange: are we not allowed to look anywhere we want for an empirically equivalent geometry?

Even stranger, they present it as a good thing:

> *"Note, though, that requiring conformal equivalence only strengthens our results. If the conventionalist cannot accommodate conformally equivalent metrics, then a fortiori one cannot accommodate arbitrary metrics; conversely, if Reichenbach's proposal fails even in the special case of conformally equivalent metrics, then it fails in the case of arguably greatest interest*." (W&M, p. 237)

> *"Again, this restriction strengthens the result. If the proposal does not work even in this special case, it cannot work in general; moreover, the special case is arguably the most interesting*." (W&M, p. 242)



To see what is going on, I think we should go back to Reichenbach's formulation.

Theorem Θ

Does Reichenbach appreciate how radical it is?

"Mathematics proves that every geometry of the Riemannian kind can be mapped upon another one of the same kind. In the language of physics this means the following" (Reichenbach, p. 32):

Theorem Θ:
Given a [Euclidean] geometry G_0 to which the measuring instruments conform, we can imagine a universal force F which affects the instruments in such a way that the actual geometry is an arbitrary geometry G, while the observed deviation from G is due to a universal deformation of the measuring instruments. (Reichenbach, p. 33)

Context of the theorem: choosing between two plausible coordinative definitions:

- Visualisability of Euclidean geometry G_0
- Simplicity of having no universal forces F=0.

But proof by intimidation: *"No epistemological objection can be made against the correctness of theorem Θ"* (Reichenbach, p. 33).

Theorem Θ it is radical! Since a**ny** metric g (on M) can then reproduce the same empirical content as **any** metric $\tilde{\mathrm{g}}$ (on M)!
→ This is the claim W&M refute.

Reichenbach (1928). *Raum und Zeit.*

Talking across purposes

Modern conventionalists are more modest: can there be peace?

- $\mathbf{1_{gen}}$: For any, $\forall_1$ g and $\forall_2$ $\tilde{g}$, both g and $\tilde{g}$ can equally represent the empirical content (Reichenbach's position).

- $\mathbf{1_{conf}}$: For any two conformally equivalent metrics, $\forall_1$ g and $\forall_2$ $\tilde{g}$, both g and $\tilde{g}$ can equally represent the empirical content (W&M's relativistic target).

Following Malament (1985), W&M take $\mathbf{1_{conf}}$ to be more representative of Reichenbach's oevre: causal relations are factual.

Since $\mathbf{1_{gen}} \rightarrow \mathbf{1_{conf}}$ Weatherall & Manchak disprove $\mathbf{1_{gen}}$ by finding a counterexample against $\mathbf{1_{conf}}$ (*modus tollens*).

That is a relevant proof, but $\mathbf{1_{gen}}$ is NOT what modern conventionalists have in mind!

- $\mathbf{2_{gen}}$: There exist distinct metric, $\exists_1$ g and $\exists_2$ $\tilde{g}$, s.t. the choice between g and $\tilde{g}$ is empirically equivalent.

This $\mathbf{2_{gen}}$ claim is still very much alive!

✗ $(\mathbf{1_{gen}})$ $\forall_1$ g ; $\forall_2$ $\tilde{g}$	$(\mathbf{3})$ $\forall_1$ g ; $\exists_2$ $\tilde{g}$?
? $(\mathbf{4})$ $\exists_1$g ; $\forall_2$ $\tilde{g}$	$(\mathbf{2_{gen}})$ $\exists_1$g ; $\exists_2$ $\tilde{g}$?

Malament (1985). ``A Modest Remark about Reichenbach, Rotation, and General Relativity.''

W&M: rigourous trading of geometry & force

A proof makes assumptions

Looking at one assumption: Conformal equivalence

Conclusion: the burden of proof…

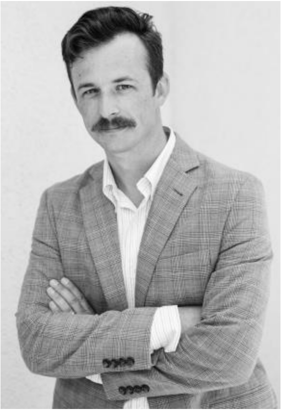
Conclusion
The burden of proof and the future …

W&M say something like:
Sure, trade-offs can always be made, but we cannot be radical skeptics. For example, we think the universal forces should be rank-2 tensors in the standard way.

D&BM reply:
All these restictions are too tight! There is a myriad of ways to construct reasonable alternative models, exploiting those assumptions!

Both search for middle ground, but do *not* meet half-way. Where does the burden of proof lie?

→ Programmatic suggestion: systematic denials of the W&M assumptions and similar proofs and disproofs.



"Ultimately, though, the attractiveness of a conventionalist thesis turns on how much one needs to postulate in order to accommodate alternative conventions. In some sense, ***one can be a conventionalist about anything****, if one is willing to postulate enough—an evil demon, say." (W&M, p. 246)*

*"****Reasonable*** *alternatives to these assumptions exist that open up geometric alternatives [...] trivial semantic conventionalism—trafficking in the conventionality of merely representational/linguistic differences of synonymous/logically equivalent content—is of little relevance to the debate." (D&BM, p. 170)*

Is general relativity safe for conventionalism? Loopholes of Weatherall & Manchak's proof

Ruward Mulder
Foundations 2023
Bristol, 8 July
UNIVERSITY OF CAMBRIDGE

Acknowledgements: many insights from this talk stem from conversations with **Neil Dewar**, **Bryan Roberts** and **Henrique Gomes**.

Literature:

Bahamonde, Sebastian; Konstantinos F. Dialektopoulos; Celia Escamilla-Rivera; Gabriel Farrugia; Viktor Gakis; Martin Hendry; Manuel Hohmann; Jackson Levi Said; Jurgen Mifsud; Eleonora Di Valentino (2021). *Teleparallel Gravity: From Theory to Cosmology*. Book available online.

Dieks, Dennis G.B.J. (1987). ``Gravitation as a universal force." *Synthese* **73**, pp. 381-397. Online access.

Dürr, Patrick & Yemima Ben-Menahem (2022). "Why Reichenbach wasn't entirely wrong, and Poincaré was almost right, about geometric conventionalism." *Studies in History and Philosophy of Science Part A* **96** (C): pp. 154–173. Online access.

Dürr, Patrick and Read, James (2023) *Reconsidering Conventionalism: An Invitation to a Sophisticated Philosophy for Modern (Space-)Times.* [Preprint] Online access.

Malament, David (1985). ``A Modest Remark about Reichenbach, Rotation, and General Relativity." *Philosophy of Science* **52** (4), pp. 615-620. Online access.

Putnam, Hilary (1974). "The Refutation of Conventionalism." *Symposia American Philosophical Association in St. Louis*. Noûs **8** (1), pp. 25-40. Online access.

Reichenbach, Hans (1928, 2014). *The Philosophy of Space & Time*. New York: Dover Publications.

Tasdan, Ufuk I & Karim P.Y. Thébault (preprint). "Spacetime Conventionalism Revisited." Online access.

Weatherall, Jim & John Byron Manchak (2014). "The Geometry of Conventionality." *Philosophy of Science* **81** (2), pp. 233-247. Online access.